# Optimization of cold-formed lipped C-section under bending using prediction equations as objective functions

Sherine Swelem, Ahmed Fahmy and Heba Ellafy[*]

*Correspondence:
eng-heba.ellafy@alexu.edu.
eg

Structural Department,
Faculty of Engineering,
Alexandria University, Bab
Sharq, Egypt

## Abstract

The use of cold-formed sections (CFS) in construction is growing quickly in spite of the difficulty facing the designer in choosing the optimum sections' proportions of any CFS in order to acquire the most economical section. In this paper, the optimization of CFS C-lipped channel beam will be carried out with respect to their flexural strength calculated using the direct strength method (DSM) given by the American specifications (AISI). The optimization will be carried out by developing an accurate and effective optimization code using particle swarm optimization (PSO). The mathematical model and the optimization technique used in this paper will help in finding the most optimum ratios between the different elements of the C-lipped channel beam that are studied in three groups. Nonlinear regression analysis is performed to obtain proposed equations that predict the flexural capacity of the cross section in two modes of failure: the local buckling mode and the distortional buckling mode. These proposed equations are used as objective functions in the optimization process as they showed that they are capable of predicting the moment capacity of the C-lipped channel effectively. It was found that the most suitable group of the three studied groups is the ratios between different elements of the cross section lies between 15 and 50%.

**Keywords:** Optimization, Cold-formed sections, Direct strength method, Particle swarm optimization, Regression analysis

## Introduction

Mainly, there are two design methods used to determine the capacity of cold-formed sections. The first method is the effective width method that uses the effective section dimensions to account for local buckling of the compression elements. The second method is the direct strength method [1] that employs strength curves for an entire section to predict the load carrying capacity. The elastic critical buckling moment can be obtained using finite element analysis or finite strip analysis (ABAQUS, ANSYS, and CUFSM). In this paper, CUFSM [2] is used to obtain the elastic critical buckling moment in the local buckling and the distortional buckling modes.

A number of researchers have proposed applying optimization methods to CFS selection and design. Tran and Li [3] had optimized the cross sectional area of

C-lipped channel beam using information obtained from a parametric study using a sort of design methods. Wie Lu [4] had optimized the capacity of Z-shape and Σ-shape CFS used as purlins calculated using Eurocode-3; the objective was to maximize the load efficiency of the purlin. Jun Ye [5] and his team applied size optimization to C-channel cold-formed beam with intermediate web stiffener at different locations to find the optimum location of the intermediate web stiffener; the section's capacity was calculated using Eurocode-3 and DSM. Jiazhen et al. [6] aimed to maximize the compressive strength of cold-formed steel columns which are not restricted to a certain shape; the final capacities of the optimized sections are found to be double the capacity of the standard cold-formed steel lipped channel; however, the optimized shapes were totally impractical. Lee et al. [7, 8] had optimized the geometry of CFS channel beams and columns; the capacity was calculated using AISI; they aimed to minimize the cross-sectional dimensions of the cross section. Seaburg and Salmon [9] aimed to find the minimum weight of cold-formed hat section that supports a given loading condition; this establishes the minimum weight design for the particular set of dimensions; the capacity was calculated using AISI. Tian and Lu [10] aimed to find the minimum dimensions of CFS channel columns with and without lips designed using the British code provisions; the studied columns had different boundary conditions. Liu et al. [11] had used DSM to design CFS column to find the optimum shape that can carry the largest critical load; they had obtained several cross sectional shapes and the maximum capacity for each shape. Moharrami et al. [12] had performed optimization to find the optimal shapes of open shaped CFS used as columns; the compressive strength was evaluated using DSM. Highly complex shapes were identified that are not suitable for practical applications. Duoc T. Phan et al. [13] used genetic algorithm to find the optimum dimensions of CFS lipped channel beam designed under the provisions of Eurocode 3. The optimized beam was used in CFS portal frames. The results showed that the optimum CFS sections exhibited significantly higher (up to 84%) ultimate capacity compared to the standard lipped channel sections with the same plate width and thickness. The structural level optimization was then carried out to obtain the optimal design solution for a long-span CFS portal frame with knee braces under serviceability limit state (SLS) and ultimate limit state conditions (ULS). Compared to conventional optimization using standard cross sections, the proposed coupled framework leaded to more cost-effective solutions (up to 20% less structural material). The results also indicated that optimizing the frame geometry and knee brace configuration could noticeably improve the structural performance and reduce the required structural weight, especially when both ULS and SLS conditions are considered. Perampalam Gatheeshgar et al. [14] had optimized CFS flexural member using particle swarm optimization; the moment capacity of the section was calculated using Eurocode 3. They were considering four CFS shapes in the optimization process. These shapes were, lipped channel beam, optimized lipped channel beam, folded-flange, and super-sigma. The results showed that, compared with a commercially available lipped channel beam with the same amount of material, the new CFS sections possessed the highest section moment capacity enhancements (up to 65%). In addition, the performance of these CFS sections, when subjected to shear and web-crippling actions, was also investigated using non-linear FE analysis.

A number of researchers have studied the parameters used in particle swarm optimization to accurately calculate these parameters to improve performance of the optimization method. M.Clerc and J. Kennedy [15] had studied PSO, and their aim was to develop the method by studding its parameters; they then introduced a set of equations that accurately calculates the parameters. R.E. Perez and K. Behdinan [16] had studied in detail the background and implementation of PSO algorithm suitable for constraint structural optimization tasks; they had introduced equations that calculates the parameters of PSO; these equations show the ability to find better optimal solutions. Zhao and Z. Ru [17] had applied PSO to the reliability analysis by combining it with first order reliability method using chaotic system, and that was done to improve the global search performance; the method was then applied in the real engineering life, and it was used to analyze the reliability of a circular tunnel in different cases.

A number of researchers had applied different techniques in regression analysis. Z.Fang, K.roy, and J.mares [18] had trained a model using deep learning in order to predict the axial capacity of cold-formed steel channel section used as a column; they have compared the output from the model by the experiments done by other researchers. They found that the results from deep learning was conservative by 9%, 6%, and 8% of stub column, intermediate column, and slender column respectively. And when they compared there results to typical shallow neural networks, they found that the performance of their network is better than it by 2%. T.gishan and K.singh [19] had performed a numerical investigation to determine the performance of cold-formed steel square hollow section. They have made finite element model, and they have validated it against experimental results from previous researches. They had also made parametric study to use it with the experimental data in comparing the results obtained from the proposed equation they had proposed; this equation is used to determine the capacity of the cold-formed columns using DSM, and it was obtained through least square regression analysis. They found that the newly proposed design equation is shown to offer conservative and reliable predictions.

## Parametric study and regression analysis

### Parametric study

In this paper, a parametric study is carried out to investigate the effect of changing the dimensions of the elements of CFS C-channel beam on the moment capacity of the beam in the local and distortional buckling modes calculated using DSM. In addition to that, it will help in choosing the best type that provide the best fit. The parametric study is performed on about 440 cross sections having different element dimensions and their corresponding flexural capacities calculated using DSM. The parametric study showed that the relationship between the dependent value which is the moment capacity and the independent variables which are the elements of the C-channel section is nonlinear.

The dimensions of the cross sections used in the parametric study are as follows: the web depth (a) ranges from 100 to 533 mm (4 inches to 21 inches) with a random increment. The flange width (b) dimension is taken as a percentage of each studied web depth that ranges from 20 to 60%, with approximately 2 mm increment (exactly 0.1 inch). The lip depth (c) is taken as a percentage of the flange width, starting from 30% and ending with 60% with a 5% increment. The thickness of the cross section is constant and is equal

to 2.667 mm (0.105 inches), the yield strength is 3515 kg/cm$^2$ (50 KSI), and the modulus of elasticity of 794,470 kg/cm$^2$ (11,300 KSI). Figure 1 shows the labels and the configuration of different elements of the C-lipped channel.

### Regression analysis

Regression analysis was developed to detect the presence of a mathematical relation between two or more variables subject to random variation and to test if such a relation is statistically significant. There are numerous types of regression models that can be used depending on the kind of data used. There are several types of nonlinear regression
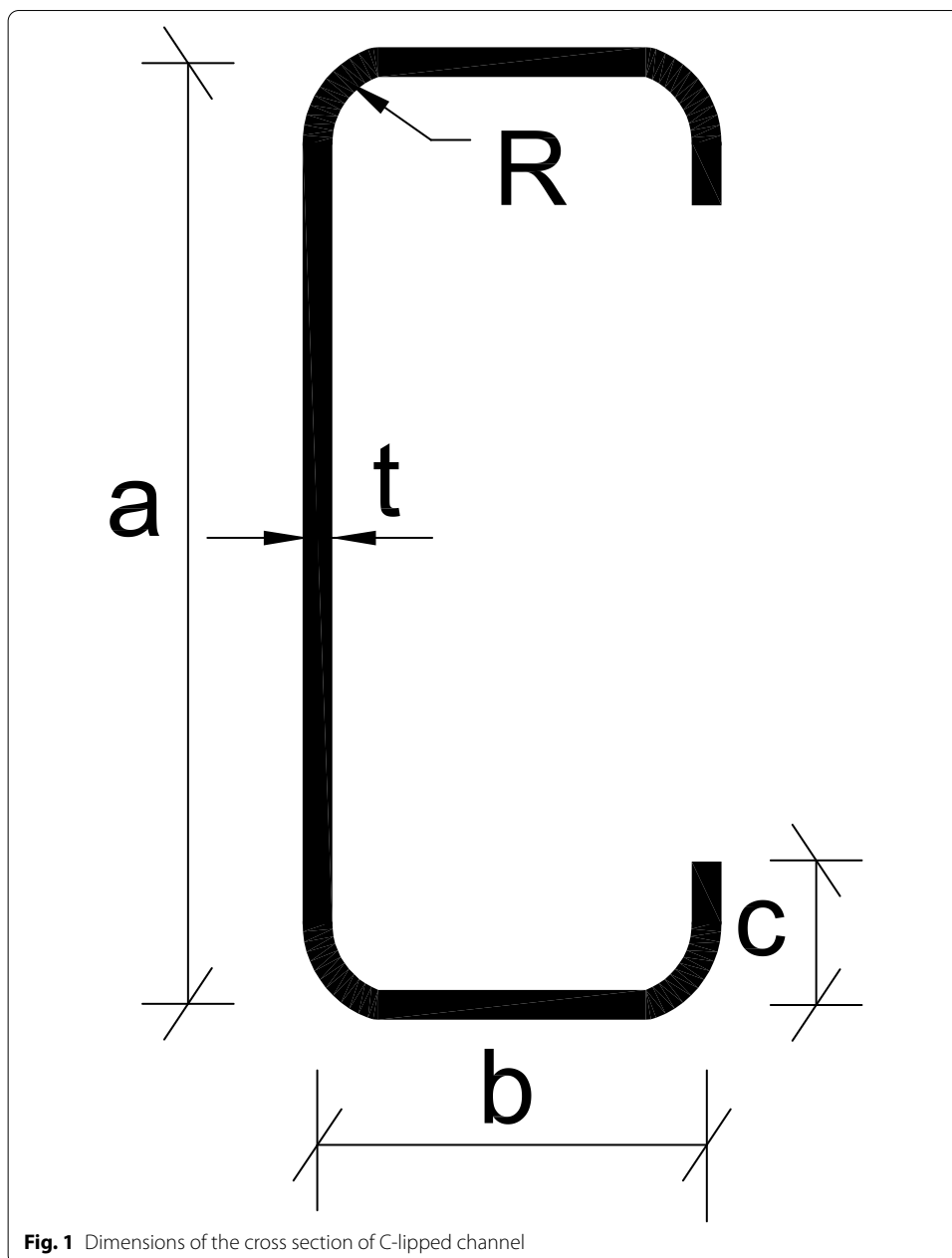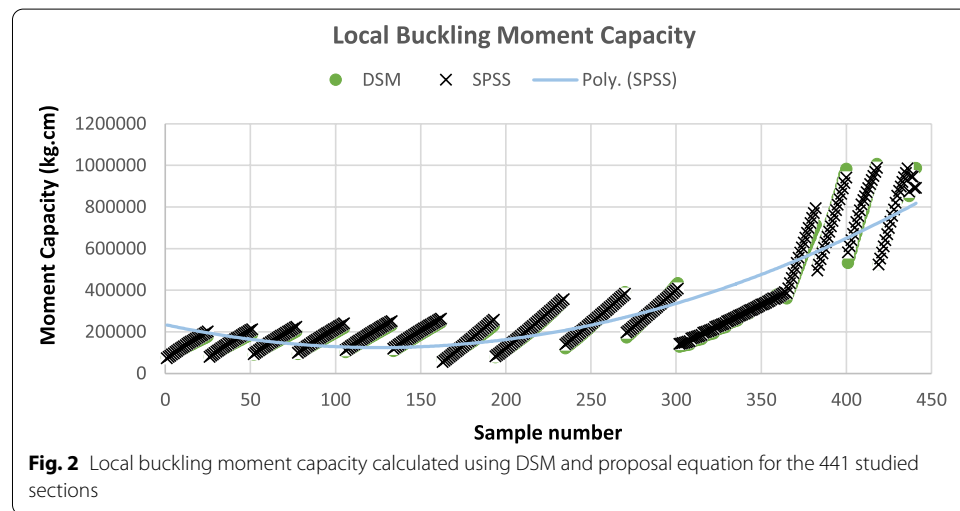


**Fig. 1** Dimensions of the cross section of C-lipped channel

**Table 1** The values of the constants used in the prediction equations for the local and the distortional buckling mode using DSM obtained using imperial units

| Design method | K | N | P | Q | S | U | V | Average error % | Maximum error % |
|---|---|---|---|---|---|---|---|---|---|
| Local buckling | − 187.4 | 20 | 1.135 | 85.2 | 0.66 | 10.1 | 1.6 | 7 | 15 |
| Distortional buckling | − 182 | 6.6 | 1.5 | 132 | 0.39 | 11.8 | 2.6 | 4 | 14 |



**Fig. 2** Local buckling moment capacity calculated using DSM and proposal equation for the 441 studied sections

analysis; each one has its strength and weakness (R.J.O Osterbaan) [20]. In this study; three types of nonlinear regression analysis are carried out using the data obtained from the parametric study that was performed previously. The three types of nonlinear regression analysis that are carried out are polynomial regression analysis, exponential regression analysis, and logarithmic regression analysis. And it was concluded that the polynomial regression analysis will provide the best prediction equations that give the least error when compared to the results calculated using DSM. A huge data base of more than 2500 moment capacities are used to obtain the polynomial equations that can predict the local and distortional buckling capacity of a laterally supported beam using software SPSS [21].

The following Eq. (1) illustrates the form of the polynomial equation that predicts the moment capacity of the C-channel cross section at constant thickness.

$$M_{cr} = K + Na^P + Qb^S + Uc^V \tag{1}$$

where $a$ is the web depth, $b$ is the flange width, $c$ the lip depth, and $K$, $N$, $P$, $Q$, $S$, $U$, and $V$ are constants that are obtained from nonlinear regression analysis using SPSS. Table 1 shows the values of the constants used in the prediction Eq. (1) for the local and the distortional buckling modes. Also, Table 1 gives the average and the maximum errors given by the prediction equations obtained from regression analysis in both failure modes. The constants shown in this table are obtained based on the imperial units (kip and inch). Figures 2 and 3 show the polynomial regression analysis curve for the studied cross sections in local and distortional buckling modes and the calculated moment capacities using DSM and prediction equations.

**Fig. 3** Distortional buckling moment capacity calculated using DSM and proposal equation for the 436 studied sections
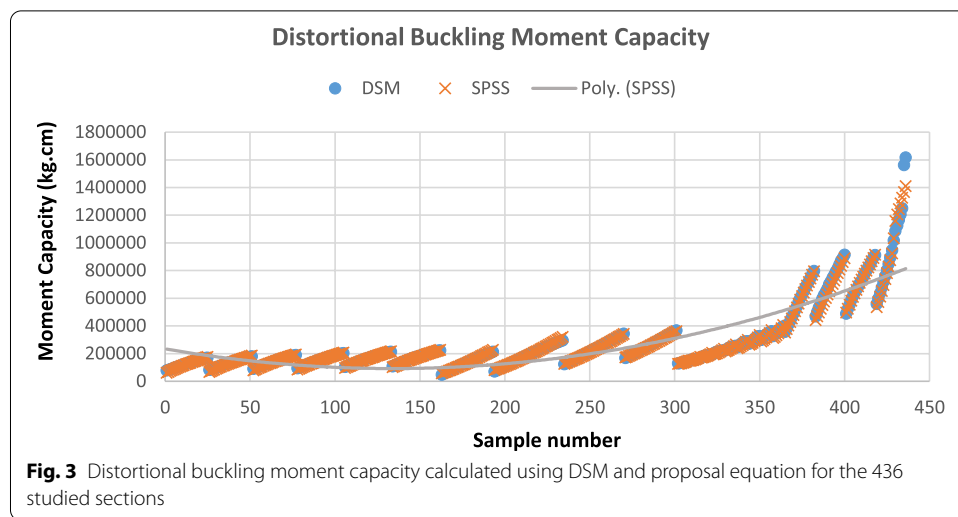
CFS are very sensitive to width to thickness ratio for the different elements of the cross section, which leads to rapid decrease in the section's capacity as this ratio increases. When studying the behavior of a wide range of dimensions for a certain element in the cross section using a constant thickness, a significant change in the capacity of the cross section will occur. While performing the regression analysis, it was found that obtaining prediction equations for a narrow range of data will be more accurate, and this is due to the complex behavior of the CFS that is affected by the increase in width to thickness ratio of the element. This is why the proportions between the different elements will be studied in three groups in the optimization model to investigate the influence of using wide and narrow ranges of elements proportions on the capacity of the section so that the best proportions between those elements can be obtained.

## Methods

Applying a suitable optimization technique to the design of cold-formed steel sections can lead to a more economical and efficient design solution. However, choosing a suitable optimization technique for CFS is a real challenge due to the complex behavior of cold-formed section and the design steps used to determine the capacity of the section. The main aim of this paper is to acquire the most efficient section proportions of the C-lipped channel beam with a fixed coil thickness using DSM by applying optimization process using particle swarm algorithm. The optimization will be performed using MATLAB [22]. The obtained proportions could help the designer choose the most economical cross section dimensions that satisfies the design requirements. The difficulty in dealing with the non-linear behavior and the complicated design procedures of cold-formed sections during optimization process is simplified by using the obtained equations from regression analysis. These equations can predict the sections' moment capacities in the local and distortional buckling modes using DSM. These equations will be used as objective functions in the optimization process.

**Fig. 4** Standard particle swarm optimization flow chart
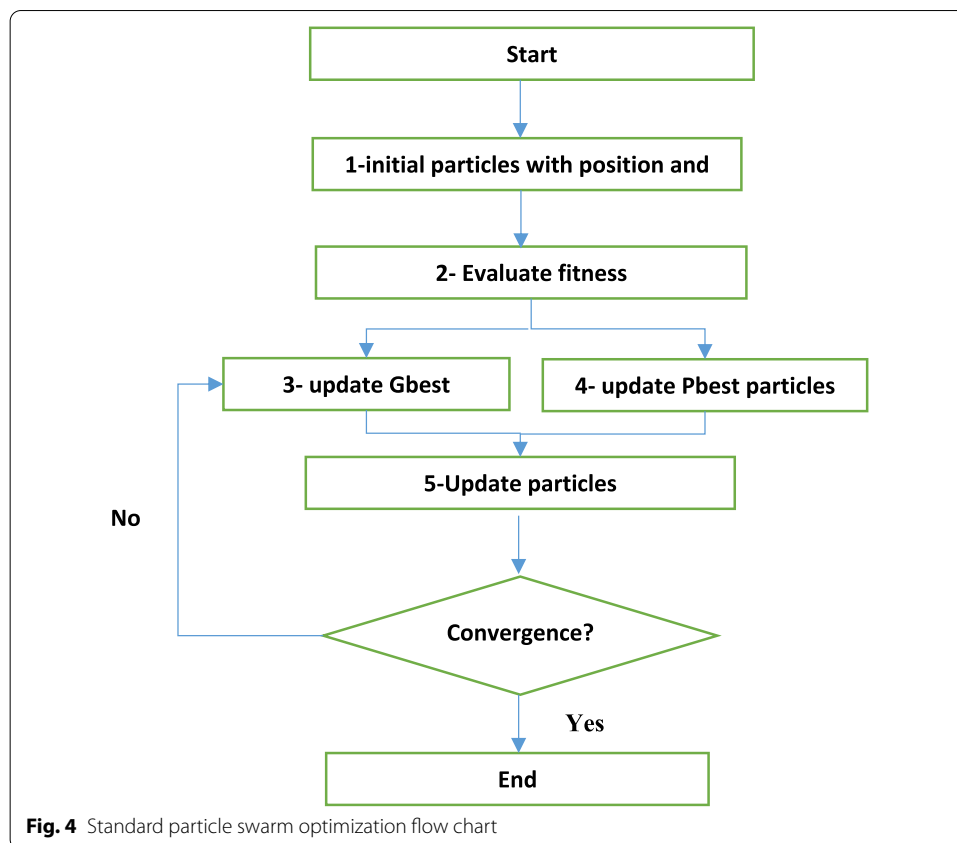
## Optimization

The optimization of cold-formed sections is defined as a constraint optimization problem. Constraint optimization is the process of optimizing an objective function with respect to some variables in the presence of constraints on those variables. The objective function could be a cost or an energy function, which is to be minimized, or a reward function or utility function, which is to be maximized. Constraints can be either equality or inequality constraints.

In this paper, the optimization is done using MATLAB by developing a practical code describing particle swarm optimization (PSO) algorithm. PSO was introduced by Kennedy and Eberhart [23]. Its efficiency in finding the optimal solutions has been measured by comparing it to genetic algorithms. The technique is based on simulating social behavior of particles "flying" through a multidimensional search space. Another mechanism introduced in PSO was proposed by Clerc and Kennedy in 2002 [15]. Also, Preze [16] suggested some new conditions to improve the convergence of a PSO problem.

## Formulation of PSO code

There are many parameters controlling the PSO algorithm performance, as they influence the mechanism of searching for the global optima. These parameters are the weight factor $w$, the two independently random variables $r_1$ and $r_2$, and the positive constant parameters called acceleration coefficients $c_1$ and $c_2$. These parameters should be

Swelem *et al. Journal of Engineering and Applied Science*    (2022) 69:49

Page 8 of 13

sensitively selected to balance between exploration and exploitation to prevent the algorithm from trapping in a local optimization. Also, the control of the weight factor will have a great impact on the efficient search for the optimum solution. The inertia weight ($w_{max}$ and $w_{min}$) should decrease linearly during the optimization that will balance the exploration and exploitation, which is a very helpful mechanism when searching for the un-known global optimum in the optimization problem. The values of the parameters $w_{max}$, $w_{min}$, and $k_{max}$ are chosen as follows: $w_{max} = 1$, $w_{min} = 0.2$, and $k_{max} = 100$. $C_1$ and $C_2$ are taken as 2.05 to ensure that the communication between the particles is at the highest level. Finally, $r_1$ and $r_2$ are random values that range from 0 to 1and they were taken in the MATLAB code as 1. The population size is taken as 50.

PSO can solve unconstrained optimization problem. As a result, the constrained optimization problem of cold-formed sections that is coded in MATLAB using PSO should be transformed into an unconstrained problem by including a penalty function. Figure 4 shows the flow chart of standard particle swarm optimization.

## Optimization model

The two prediction equations obtained from the nonlinear regression analysis are used as the objective functions in the optimization problem. During the optimization process; the objective function that describes the flexural capacity of the beam will be maximized.

The optimization process is repeated for six times; the first three optimization process are for the local buckling mode, where the prediction equation calculating local buckling capacity is used as the objective function and the constraints differs for groups A, B, and C. The other three optimization processes are for the distortional buckling mode for the three groups, and the objective function is the prediction equation which calculates the distortional buckling capacity.

### Description of the objective function

As mentioned before, the objective functions are the two prediction equations obtained from the regression analysis. These equations calculate the moment capacity of the CFS C-section in the local and the distortional buckling modes using DSM (see Eq. 1 in the "Regression analysis" section).
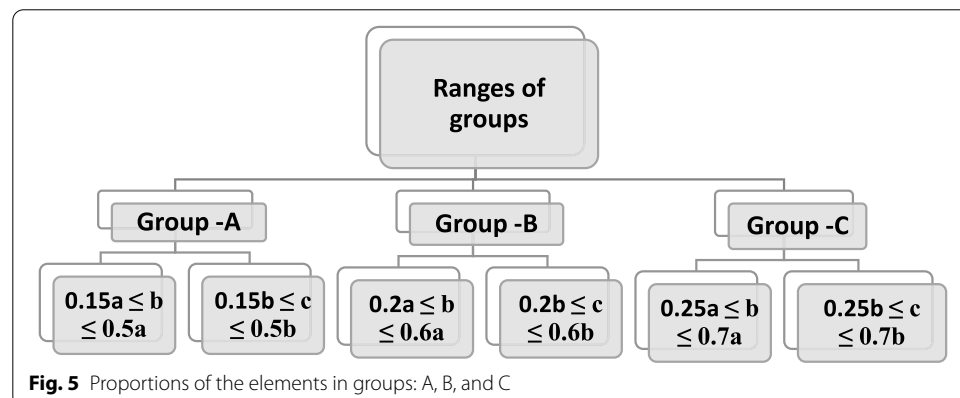


**Fig. 5** Proportions of the elements in groups: A, B, and C

Swelem *et al. Journal of Engineering and Applied Science*      (2022) 69:49

Page 9 of 13

**Table 2** Recommended width to thickness ratio (w/t) in AISI

| Element | Web | Flange | Lip |
| --- | --- | --- | --- |
| w/t for different elements | **200** | **90** | **14** |

**Table 3** The maximum dimensions of the different elements of C-section considering the width to thickness ratio as specified by AISI at a constant thickness of 2.667 mm (0.105 inches)

| Element | Web depth | Flange width | Lip depth |
| --- | --- | --- | --- |
| Maximum dimensions (mm) | **533 (21 inches)** | **253 (9.95 inches)** | **44 (1.72 inches)** |

### Description of the variables

The variables used in the optimization model are the web depth (a), the flange width (b), and the lip depth (c), respectively.

### Description of the constraints

The constrains are linear inequalities and they describe the ratios between the elements of the cross section. Figure 5 shows the three studied groups A, B, and C. The relationship between the different elements in group A ranges from 15 to 50% having a difference of 35%. As for group B, the range is 20 to 60% having a difference of 40%, and in group C, the range starts from 25% and ends with 70% having a difference of 45%. The results obtained from the three groups cover more area including the optimum range of proportions between the elements of the cross section and the best dimensions for the flange and the lip elements. The effect of using these three groups on the capacity of the CFS is considered and applied through the optimization process by using these proportions as constrains in the mathematical model during the process.

Upper and lower limitations are set to ensure that the obtained results are practical considering the fabrication, the construction, and design perspective. The lower bound is set as follows:

  I.  Flange width (b) should exceed 30 mm (1.2 inches) to permit sufficient space for the connection between the flange and the metal decking.
 II.  The lip depth (c) should be at least 10 mm (0.4 inches), so that it can be rolled or brake-pressed.
III.  The minimum web depth (a) should be 100 mm (4 inches), so that the connection between the web and any other part can be easily settled.
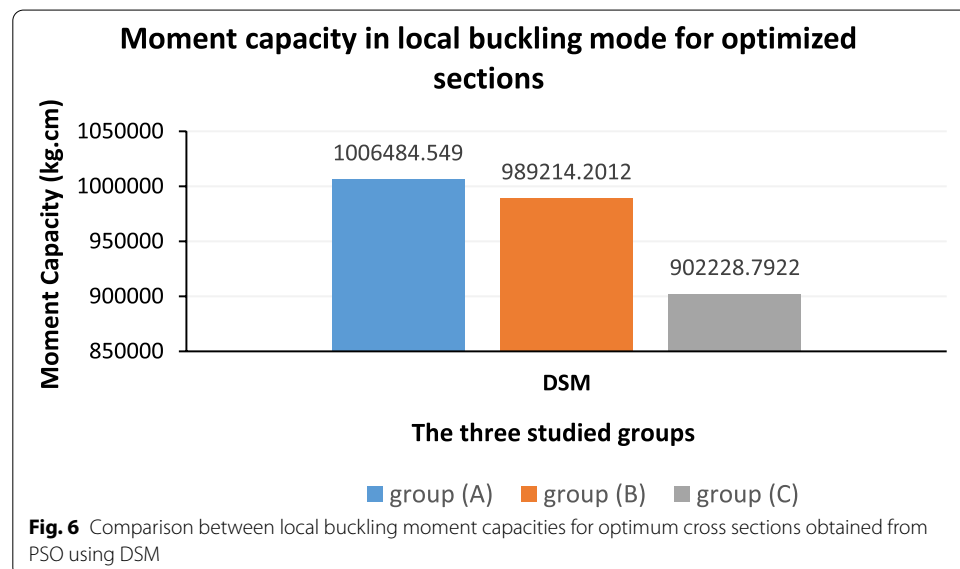
The upper bound is calculated using the width (*w*) to thickness (*t*) ratios of different elements of the cross section following AISI. Table 2 lists the width to thickness ratio as specified by AISI. Table 3 shows the maximum dimensions of the different elements of the C-section considering the width to thickness ratio as specified by AISI having a constant thickness of 2.667 mm (0.105 inches).
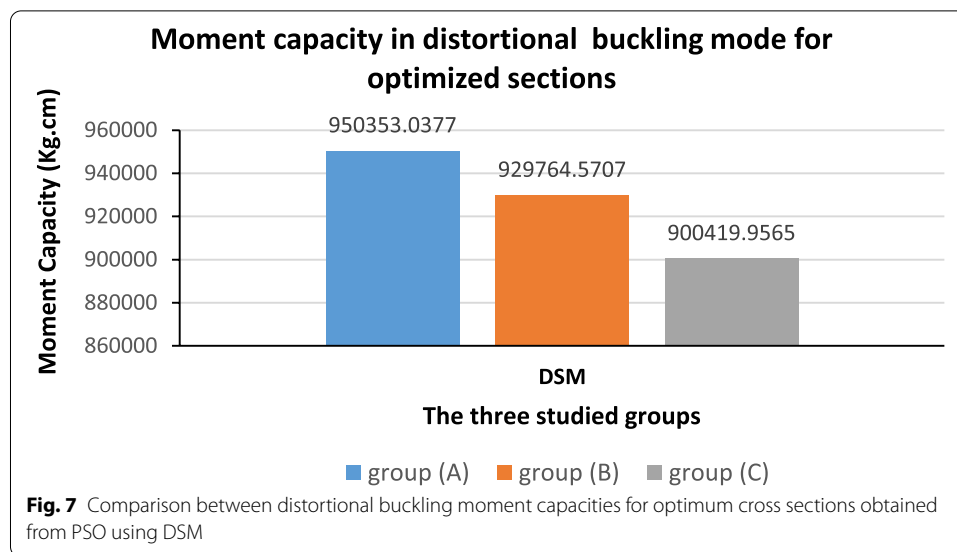
Swelem *et al. Journal of Engineering and Applied Science*        (2022) 69:49

Page 10 of 13

**Table 4** Optimization results in the local buckling mode using proportions of groups A, B, and C

| | Optimum dimensions | | | Moment capacity using the original design method (kg.cm) | Moment capacity using proposed equations (kg.cm) | Error % | Area of section (mm²) |
|---|---|---|---|---|---|---|---|
| | a mm | b mm | c mm | | | | |
| Group A | 533 (21 inches) | 253 (9.95 inches) | 44 (1.72 inches) | 1,006,485 (873.59 kip. inch) | 988,869 (858.3 kip. inch) | +1.74 | 2948.381 |
| Group B | 533 (21 inches) | 218 (8.6 inches) | 44 (1.72 inches) | 989,214 (858.6 kip. inch) | 947,841 (822.69 kip. inch) | +4.18 | 2767.736 |
| Group C | 533 (21 inches) | 175 (6.88 inches) | 44 (1.72 inches) | 902,229 (783.1 kip. inch) | 892,217 (774.41 kip. inch) | +1.1 | 2535.479 |

**Table 5** Optimization results in the distortional buckling mode using proportions of groups A, B, and C

| | Optimum dimensions | | | Moment capacity using the original design method (kg.cm) | Moment capacity using proposed equations (kg.cm) | Error % | Area of section (mm²) |
|---|---|---|---|---|---|---|---|
| | a mm | b mm | c mm | | | | |
| Group A | 533 (21 inches) | 253 (9.95 inches) | 44 (1.72 inches) | 950,353 (824.87 kip. inch) | 911,653.2 (791.28 kip. inch) | −4.24 | 2948.381 |
| Group B | 533 (21 inches) | 218 (8.6 inches) | 44 (1.72 inches) | 929,764.6 (807 kip. inch) | 918,243.3 (797 kip. inch) | −1.2 | 2767.736 |
| Group C | 533 (21 inches) | 175 (6.88 inches) | 44 (1.72 inches) | 900,420 (781.53 kip. inch) | 901,537.5 (782.5 kip. inch) | +0.12 | 2535.479 |



**Fig. 6** Comparison between local buckling moment capacities for optimum cross sections obtained from PSO using DSM

**Fig. 7** Comparison between distortional buckling moment capacities for optimum cross sections obtained from PSO using DSM

## Results and discussion

The results obtained from the optimization process using different constraints are listed in Tables 4 and 5 and Figs. 6 and 7. Table 4 and Fig. 6 show that the minimum and maximum local buckling capacities are given by group A and group C respectively. The variance in the moment capacities is more than 10% more for group A. The results show that using a narrow range of section proportions gives more capacity than wide ranges. Proportions in group A are the best as it gives the highest capacity. Table 4 also shows that the maximum area of the cross section is given by the section obtained from group A proportions, while the minimum area of the cross section is given by the section obtained from group C proportions. That means group C gives 6% less weight or cost than group A.

Table 5 and Fig. 7 show the results from optimization for the distortional buckling mode, and the results were exactly the same as for the local buckling mode. These results are as follows: the minimum and maximum distortional buckling capacities are given by group A and group C respectively; the variance in the moment capacities is about 5% more for group A. The results prove that narrow range of section proportions gives more capacity from wide ranges, and group A proportions is the best as it gives the highest capacity. Table 4 shows that the maximum area of the cross section is given by the section obtained from group A proportions, while the minimum area of the cross section is given by section obtained from group C proportions. That means group C gives 6% less weight or cost than group A. From the previous results of the optimization process, it is concluded that as the range between the proportions of the elements decreases, as the capacity of the section increases.

## Conclusions

A parametric study is performed using a wide range of C-lipped channel cross sections with different dimensions and constant thickness of 0.105 inches. The moment capacities for these cross sections are obtained in the local and the distortional buckling

Swelem *et al. Journal of Engineering and Applied Science*        (2022) 69:49

Page 12 of 13

modes. They are calculated using the direct strength method (DSM). The number of the studied cross sections are about 440 sections and the number of corresponding moment capacities in the two failure modes are more than 2500 moment capacities. The data obtained from the parametric study is used for nonlinear regression analysis to obtain prediction equations that can predict the moment capacities of the different sections in the local and the distortional buckling modes with high accuracy.

Optimization process is performed for the C-lipped channels using particle swarm optimization (PSO) algorithm. A modified and developed optimization code is used in MATLAB. The optimization process was done with respect to the flexural capacity of the C-lipped channel beam using the prediction equations obtained for the local buckling mode and the distortional buckling mode as an objective function. The constraints are the proportions between the different elements of the C-lipped channel. The ratio between the elements are classified in three main groups: A, B, and C. Using the proportions between the elements as constraints in the optimization problem helps in determining the best one for the designer.

The use of the prediction equations as objective functions in the optimization procedures facilitates the process, because there is no need to insert the complicated and many design procedures of CFS as a black box in MATLAB. The use of regression analysis to obtain objective functions that can be used in the optimization process is a powerful tool that could be tested and validated in other fields in the future.

From the three suggested groups of proportions A, B, and C, the maximum capacity in the local and the distortional buckling modes are given by group A. These proportions are as follows:

$$0.15 \leq (b/a) \leq 0.5$$

$$0.15 \leq (c/b) \leq 0.5$$

As a result, using a narrow range of ratios between the different elements of the cross section under bending in the local and the distortional buckling modes could help the designer to acquire the best proportions that leads to the maximum section capacity.

### Limitations of this study

In this paper, the shear and serviceability constraints are not applied. It would be helpful to study other section configurations like Z and sigma sections.

Swelem *et al. Journal of Engineering and Applied Science*        (2022) 69:49

Page 13 of 13

## Declarations

**Competing interests**

The authors declare that they have no competing interests.

## References

1. AISI (2016) Supplement 2016 to the North American Specification for the Design of Cold-Formed Steel Structural Members, Appendix 1, Design of Cold-Formed Steel Structural Members Using Direct Strength Method. American Iron and Steel Institute, Washington, D.C., pp S100-16
2. Shafer B.W, Adany S (2006) "Buckling analysis of cold formed steel member using CUFSM: conventional and constrained Finite Strip Methods", Eighteenth international specialty conference on cold formed steel structures Orlando, Florida, U.S.A
3. Tran T, Li L-Y (2006) "Global optimization of cold-formed steel channel sections". Thin Walled Structures 44(399):406 School of Engineering and Applied Science, Aston University, Birmingham, UK
4. Lu W (2003) "Optimum design of cold-formed steel purlins using genetic algorithms" ph.D Thesis. Helsinki University of Technology, Department of Civil and Environmental Engineering, Laboratory of Steel Structures, Espoo
5. Ye J, Hajirasouliha I, Becque J, Eslami A (2016) "Optimization design of cold-formed using particle swarm optimization method". J Construct Steel Res 122:80–93 Department of Civil Engineering and structural Engineering University of Sheffield Sheffield, UK
6. Leng J, Guest JK, Shafer B.W (2011) "Shape optimization of cold formed steel columns", Hopkins University, USA. J Thin Walled Structures 49(12):1492–503
7. Lee J, Kim SM, Park HS (2005) "Optimum design of cold formed steel channel beams using micro-genetic algorithm". Eng Struct 27:17–24
8. Lee J, Kim SM, Park HS (2006) Optimum design of cold formed steel columns using micro-genetic algorithm. Thin-Walled Struct 44:925–960
9. Seaburg PA, Salmon CG (1971) Minimum weight design of light gage steel members. J Struct Div ASCE 97(1):203–222
10. Tian YS, Lu TJ (2004) Minimum weight of Cold-formed steel sections under compression. Thin Walled Struct 42(4):515–532
11. Liu H, Igusa T et al (2004) Knowledge based global optimization of cold formed steel columns. Thin Walled Structures 42(6):785–801
12. Moharami M, Louhghalam A (2014) Optimal folding of cold formed steel cross sections under compression. Thin Walled Structures 76:145–156
13. DT Phan, SM Mojtabaei, I Hajirasouliha, J Ye, JB Lim. 2019. "Coupled element and structural level optimization framework for cold-formed steel frames". Journal of Constructional Steel Research. 168. 105867. ISSN 0143-974X
14. Gatheeshgar P, Poologanathan K, Gunalan S, Nagaratnam B, Tsavdaridis KD, Ye J (2020) "Structural behaviour of optimized cold-formed steel beams" Verlag für Architektur und technische Wissenschaften GmbH & Co. KG, Berlin. Steel Construction 13:294–304
15. Clerc M, Kennedy J (2002) The particle swarm - explosion, stability, and convergence in a multidimensional complex space. IEEE Trans Evol Comput 6:58–73
16. Perez RE, Behdinan K (2007) "Particle Swarm approach for Structural design optimization", University of Toronto, Canada ELSEVIER, computer and structures
17. Hongbo Z, Xu C (2014) "Reliability Analysis using chaotic Particle Swarm Optimization". Jiaozuo, china: Wiley online library, Department Civil Engineering, University of Henan Polytechnic
18. Fang Z, Roy K (2021) Deep learning based axial capacity prediction for cold formed steel sections using Deep Belief Network. J Elsevier 33:2792–2802 Department Civil Engineering, University of Auckland, New Zealand, 2021
19. Gishan T, Singh K (2021) Design of perforated cold formed hollow stub columns using direct strength method. J Elsevier 168:108265 Department Civil Engineering, institute of technology Jodhpur, India, 2021
20. Oosterbaan RJ (1994) "Frequency and regression analysis in: H.P. Ritzema (ed.), Drainage Principles and Applications." International Institute for Land Reclamation and Improvement (ILRI), Wageningen
21. IBM Corp. Released 2015. IBM SPSS Statistics for Windows, Version 23.0. Armonk, NY: IBM Corp
22. MATLAB and Statistics Toolbox Release 2013, The MathWorks, Inc., Natick, Massachusetts, United States
23. Kennedy J, Eberhart R (1995) "Particle swarm optimization". Proceedings of IEEE International Conference on Neural Networks. IV