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An automated optimized design of practical post-tensioned slabs

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Abstract

In this paper, an efficient methodology that could be implemented within any analysis software is developed to automate the design process of post-tensioned slabs. Optimization techniques are used to compute the necessary number of strands. The present methodology is based on computing an innovative influence matrix that contains the effect of each strand on stresses at a set of control points (points of peak stresses). This matrix will be used in the optimization avoiding reanalysis in each optimization iteration. The sum of the squares of the difference between the allowable stress and the computed stresses at all control points is considered an objective function. The optimization is then carried out under the constraints that the stresses at each control point should not exceed the allowable limit and to ensure that the number of strands is an integer value. Alternatively, the proposed methodology could be used as a value-engineering tool. Practical examples are presented to demonstrate the efficiency of the developed methodology.

Highlights

- Automated design tool of practical PT slabs independent of designer experience.
- Ability to perform value engineering tasks to obtain optimum number of cables in practical PT slabs.
- Practical use of optimization methods in structural design automation tool.

Keywords: Automated design, Structural optimization, Post-tensioned slabs, Value engineering

Introduction

Post-tensioning is used in various structural elements, such as slabs and beams, as it has many advantages. It allows architectural engineers to work freely with their designs to provide large spans. Moreover, it provides thinner slabs, less cracks, and lighter structures as it provides material reduction in columns and foundations. Generally, it saves about 9% to 19% of the slab cost and about 7% to 14% of the skeleton cost for spans between 6.0 and 10.0 m [1] while allowing for speed in the construction process. These advantages encourage engineers to consider post-tensioned (PT) slabs as an alternative to other types of slab construction such as solid and flat slabs.

Many software tools are available to design PT slabs. Among them are SAFE2000 [2], ADAPT-Builder [3], RAM Concept [4], and PLPAK [5]. Traditional designs using such packages are carried out in an iterative way. Finally, the computed quantities are not optimum as they are usually overdesigned by at least 15% as will be demonstrated in Section (4) this paper.

Several researchers have studied the analysis and design of PT slabs. Ellobody et al. [6] studied the behavior of un-bonded one-way simply supported PT slab in fire conditions using nonlinear finite element (FE) models. Hanaa et al. [7] studied the behavior of two-way PT slab using the RAM Concept software [4] with different effects as changing concrete strength, thickness of the flat slab, and jacking forces. Al Rawi et al. [8] investigated the behavior of PT slabs under impact load. Rashed [9] presented a boundary element structural analysis of PT slabs.

Many other researchers had applied optimization techniques to PT structures. Hassan et al. [10] built FE analysis coupled with optimization technique to determine optimum PT cable forces in bridge beams under the dead load to achieve both minimum deflection and uniform bending moment (BM) distribution. Mohammed et al. [11, 12] developed two FE models of bonded and un-bonded one-way PT slab. The objective function in case of the un-bonded cables model was the total weight of PT cables, whereas in case of the bonded cables model, the objective function was the total strain energy. The considered variables were the area of PT cables, initial stress, and tendon eccentricity. The constraints were the concrete normal stresses, steel tendon stress, and mid-span deflection of PT slab. It should be noted that the work in [11, 12] is not applicable for typical PT slabs as it was developed only for one-way simple slabs. Sarkisian et al. [13] used topology optimization to determine the optimal load paths using FE model. The tendons were arranged along the obtained optimal load paths. The quantities of PT reinforcement were reduced by about 25% relative to the actual design. Three case studies were considered in [13]. However, the obtained cable profiles were theoretical and not practical. Galv et al. [14] developed numerical optimization model for one-way PT slab with un-bonded cables. The objective function was the minimization of material and labor costs. Variables were slab thickness, tendon eccentricity, and number of tendons. The work in [14] is not applicable for practical problems as it treats only one-way PT slabs. Yousif and Saka [15] demonstrated design optimization algorithms for PT slabs. The objective function in [15] was the total cost of the slab, which is needed to be minimized. The variables were as follows:

1) PT slab thickness
2) Number of tendons and their section along the two directions
3) Additional reinforcements at the top and bottom face of the slab and the punching shear reinforcements
4) Columns cross-sectional dimensions, number, and diameter of reinforcements

The considered optimization frameworks in [15] were based on Artificial Bee Colony (ABC), Particle Swarm Optimization (PSO), and Beetle Antenna Search (BAS). These techniques were applied to a small slab with regular span and column arrangement. According to the building code requirements [16], a suggested slab thickness is provided to ensure safety against deflection limit state. The given design thickness according to building code provides the minimum number of strands as demonstrated in [17]. Therefore, in practice, slab thickness should not be considered a design variable in the optimization process. The other variables, such as additional traditional reinforcement, are usually placed at minimum quantity, if a reasonable PT pattern is used. Column positions are always postulated by architecture engineers. As such, the only major variable considered in [15] is the number of strands, which should be accounted for any practical design. The examples solved in [15] are simple slabs with no configuration of relevance in practice.

As is clear from the above literature review, most previous research did not account for design factors that are important in practical applications. Others considered theoretical cable profiles that cannot be applied in practice. Moreover, according to designers, the major important variable in PT slab optimization is the number of strands which directly contributes to the total weight of PT cables and consequently the cost.

In this paper, an efficient optimization methodology is developed to automate the design process of PT slabs. The proposed methodology, as will be demonstrated in the examples section "Numerical examples", is applicable for typical slab configurations used in practice and can be coupled with any analysis software. The objective function is set and minimized to achieve the optimum number of strands. The proposed technique could be also used as a value-engineering tool to minimize the number of strands and cables in an already designed PT slab. It was demonstrated that using the proposed methodology ensures a lean process as well as being independent of the designer's experience. Moreover, it ensures savings for practical slabs up to 30% in material cost. It has to be noted that the automation herein is performed on the design process, not in the modeling of the problem as demonstrated in Fig. 1.

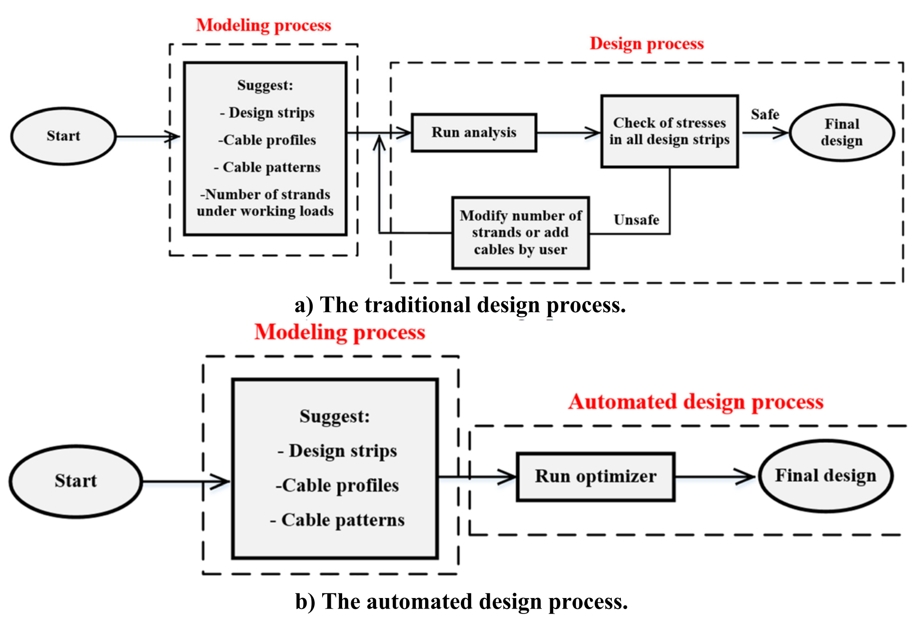


Fig. 1 The traditional verses the automated design process

Optimization methodology

In this section, the proposed optimization methodology and process are presented. The proposed objective function is defined as the summation of the squares of the differences between the allowable stress and the actual tensile stress. This function is computed under the desired load combination such as the working loads at the control points, which are considered points of peak tensile stresses. Two types of constraints are considered; the first is that the stress at each control point should not exceed a threshold limit (the allowable stress limit), and the second is that the computed number of strands in each cable must have an integer value.

The steps of the proposed optimization can be summarized as follows:

1- As in traditional design steps, design strips are first defined.
2- A single cable with a single strand at the midline of each design strip is placed. The profile of cables is defined, as shown in Fig. 2, according to bending moment along a strip at the midline of the design strip.
3- Control points are selected at points of maximum top and bottom stresses along the two directions.
4- An influence matrix $[F]_{m\times n}$ is then built as shown in Fig. 3, where n represents the total number of control points and m represents the total number of cables. The influence matrix elements are computed as the values of the stresses at control points when applying a jacking force P at a certain cable and setting the force in each of the other cables to be zero. Force P is the jacking force of one strand according to the design code. It has to be noted that this influence matrix is used in the optimization process avoiding carrying out reanalysis during the optimization iterations.
5- At each control point j, the following function V_j is defined:

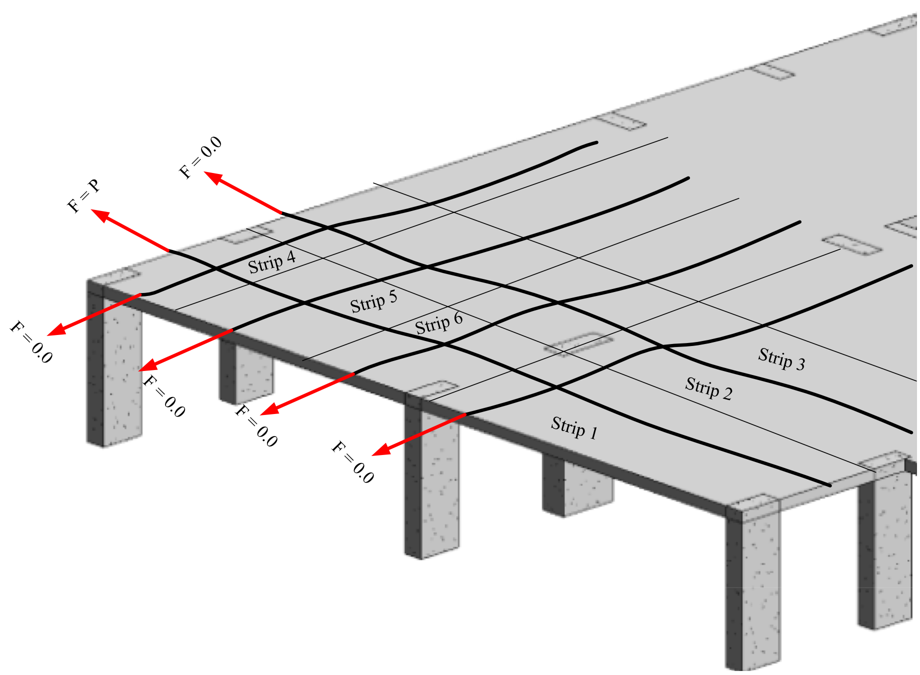


Fig. 2 Illustrative example showing cables and design strips

Influence matrix $[F]_{i \times j}$ =

	CP_1	CP_2		CP_j
n_1	F (1,1)	F (1,2)	…	F (1, j)
n_2	F (2,1)	F (2,2)	…	…
	…	…	…	
n_i	F (i,1)	…	…	F (i, j)

Fig. 3 The influence matrix [F] for the considered slab

$$V_j = \sigma_j - [\sigma_{all} + \sum_{i=1}^{m} n_i \times F(i, j)] \tag{1}$$

where V_j is the difference between the stress due to the working loads σ_j and the sum of the allowable stress σ_{all} and the cable effect $\sum_{i=1}^{m} n_i \times F(i, j)$. It should be noted that V_j is computed at all control points regardless of whether it represents top or bottom maximum stress and whether it is along X or Y direction. n_i is the number of strands in cable number i. The objective function V_t is defined as the summation of squares of V_j functions at all control points, as follows:

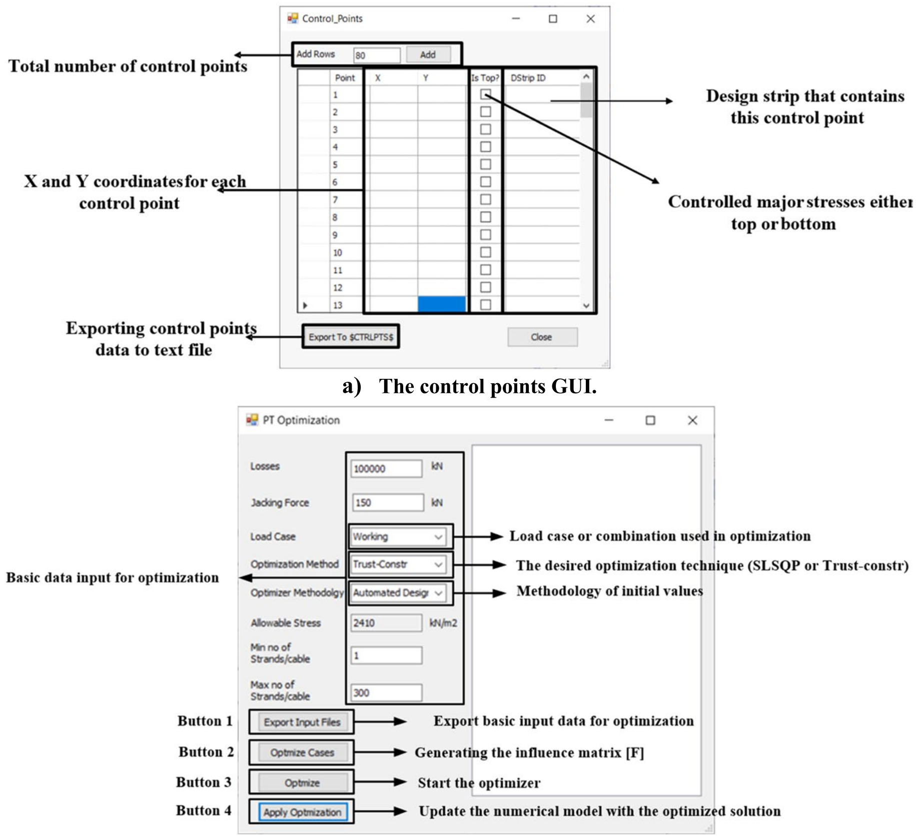
$$V_t = \sum_{j=1}^{n} {V_j}^2 \tag{2}$$

Hence, a convex optimization scheme is executed to minimize the objective function V_t to compute the optimum number of strands n_i in each cable or design strip.

The proposed optimizer tools

In this section, the proposed idea is implemented into a computer code. Two graphical user interfaces (GUIs) are developed. The two GUIs can be linked to any structural analysis software. These GUIs are the control points GUI, and the optimization GUI as demonstrated in Fig. 4. Control points GUI is used to select the control point's data, whereas the optimization GUI is for input of the basic PT data. The optimization GUI is working via four stages:

1- The first stage is to export the basic data (Button 1: optimization GUI, Fig. 4b) such as losses, jacking force, the name of load case/combination on which the optimization process is carried out (depending on its stresses result), the optimization method, the optimizer methodology, allowable stress, and minimum and maximum allowable number of strands/cable. It has to be noted that the user can start the optimization process using single strand in each cable. This is in case he is carried out the design of the slab. Alternatively, the user can use the number of strands obtained from previous design, in case of carrying out value engineering process.



b) The optimization GUI.

Fig. 4 The developed GUIs

2- The second stage is "optimize cases" (Button 2: optimization GUI, Fig. 4b) to compute the innovative influence matrix, which will be used in the optimization process without retuning to the analysis software.

3- The third stage is "optimize" (Button 3: optimization GUI, Fig. 4b), to perform the optimization process.

4- The fourth stage is "Apply optimization" (Button 4: optimization GUI, Fig. 4b) to update the numerical model with the final optimized number of strands.

It has to be noted that the optimization problem uses the python library Scipy.optimize.minimize [18]. Two algorithms of optimization are considered:

i. The "Trust-constr" algorithm
ii. The "Sequential Least Squares Programming SLSQP" algorithm

The "Trust constr" method is a trust region algorithm for solving constraint optimization problems. This is the most universal minimization algorithm with constraints implemented in SciPy, and the most suitable for large-scale problems [18]. The "SLSQP," on the other hand, is a gradient-based method for solving nonlinear optimization problems with constraints, in which constraints and the objective function are continuously differentiable [19]. The developed optimization GUI can switch between the two

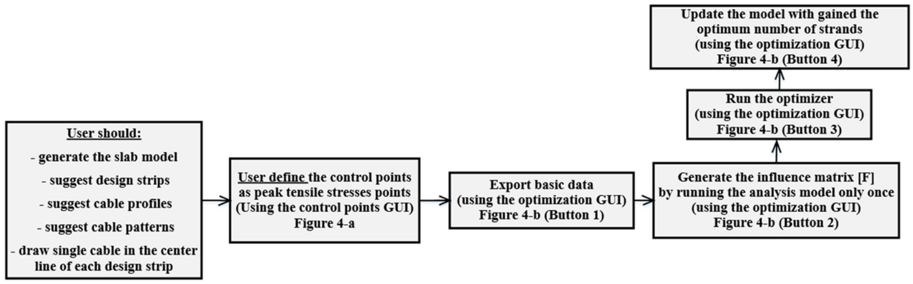


Fig. 5 The overall proposed design process flow chart

Table 1 The cables' material properties

Modulus of elasticity	1.95×10^8 kN/m^2
Yield stress	1.67×10^6 kN/m^2
Ultimate stress	1.86×10^6 kN/m^2
Area of one strand	98.71 mm^2
Seating losses	6 mm
Friction coefficient	0.2
Wobble coefficient	0.0033

algorithms according to the user input. The output total number of strands in each design strip is uniformly redistributed within the design strip to satisfy the design code requirements. In this paper, and without losing the generality, the PLPAK software [5] is used as analysis software and the updating is carried out via text files. Figure 5 demonstrates the overall proposed design process flow chart, and the user intervention.

Numerical examples

As mentioned earlier, the developed tools can be linked to any analysis software package. The PLPAK software, which is used, inherits the capability of the boundary element method in considering no internal discretization together with modelling the actual geometry of the supporting elements, leading to more accurate cable placements as demonstrated in [9]. In this section, three practical examples are solved using the proposed technique to demonstrate its efficiency and validity. The first example demonstrates the use of the developed tools to automate the design process of PT slab. The second and the third examples demonstrate the developed tool in carrying out value engineering tasks. In which the optimized design is compared to the traditional design, that commonly carried out in practical design companies.

In the three examples, Young's modulus of the concrete is 260×10^5 kN/m^2, and the Poisson's ratio is 0.2. The weight per unit volume of concrete is 25 kN/m^3, and the characteristic strength of the concrete is 35 N/mm^2. Based on the stresses results, the load combination to be optimized is the working load combination (the summation of all assigned loads). The allowable stress is assumed to be 2601 kN/m^2. Table 1 shows the material properties of the cables.

In the examples, the term "working load" denotes the summation of all assigned loads, and the term "service load" denotes the working loads together with cable effects.

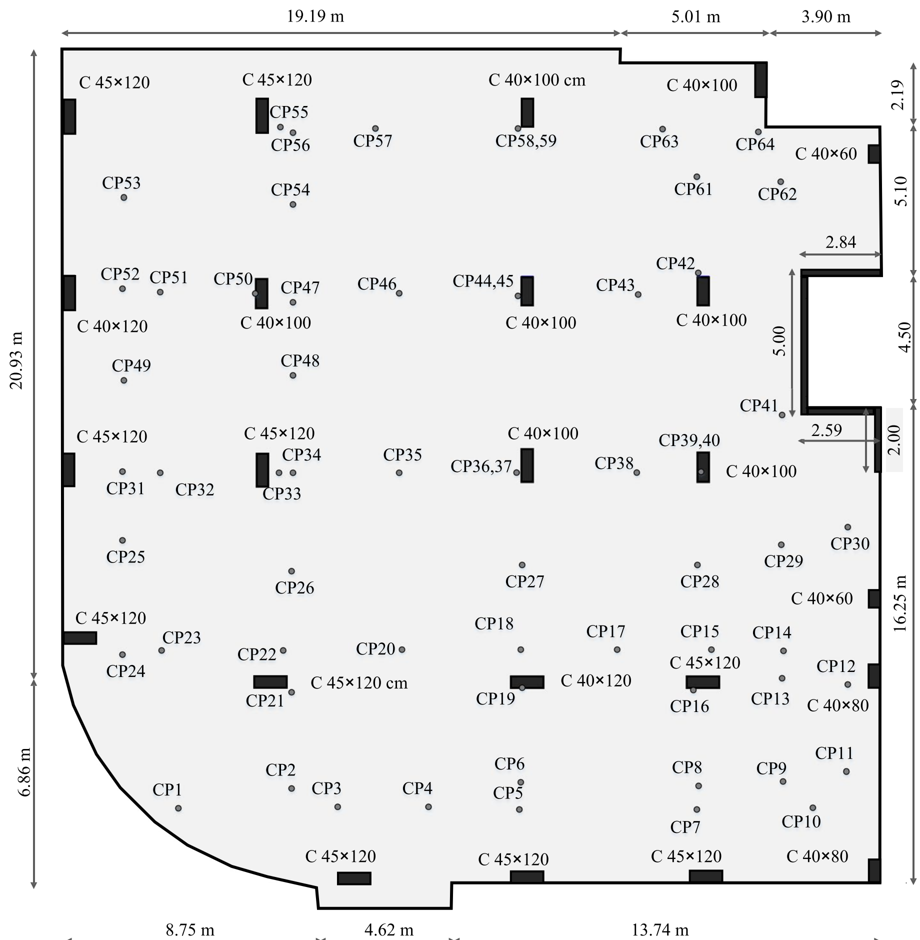


Fig. 6 Layout and dimensions and control points in Example 1

Practical example 1

The slab shown in Fig. 6 is considered. It has a thickness of 0.26 m. The assigned load is 5 kN/m^2 for superimposed dead load and 3 kN/m^2 for live load.

The slab is divided into 5 and 6 design strips along X and Y directions, respectively. Figure 7 represents the numbering system for the cables and the design strips. Sixty-four control points are chosen (Fig. 6). The output number of strands in each cable is shown in Table 2. It can be seen that results using the two-optimization algorithms are identical after rounding to integer values. But the values are presented in Table 2 are left as real values just to demonstrate that each optimization method has a different output. Figure 8 shows the final stresses acting on one chosen design strip after assigning the optimum number of strands. The final stresses are safe and optimum in all strips as they are lower than the allowable stress. It has to be noted that C 45×120 denotes a column with dimensions 45 cm × 120 cm.

Practical example 2

The purpose of this example is to demonstrate the use of the proposed tools to carry out value engineering of already designed PT slab. The considered slab is shown in Fig. 9. The slab has a thickness of 0.22 m. The considered load is 5 kN/m^2 for

Fig. 7 Design strips and cables numbering in Example 1

Table 2 The optimum number of strands in each cable in Example 1

Cable ID	Optimization method		Computed number From either method (after rounding)
	Trust-constr	SLSQP	
C_1	18.79044578	18.72398317	19
C_2	15.80080488	15.75532604	16
C_3	15.41700084	15.3667356	16
C_4	15.37253143	15.31944377	16
C_5	13.16347224	13.10754237	14
C_6	6.79078827	6.7568517	7
C_7	14.59157746	14.54355378	15
C_8	11.1973595	11.1755662	12
C_9	11.92034224	11.80695017	12
C_{10}	1.00000095	1.33950296	1
C_{11}	1.00000284	1.00054475	1
Total weight			**2720.2 kg**

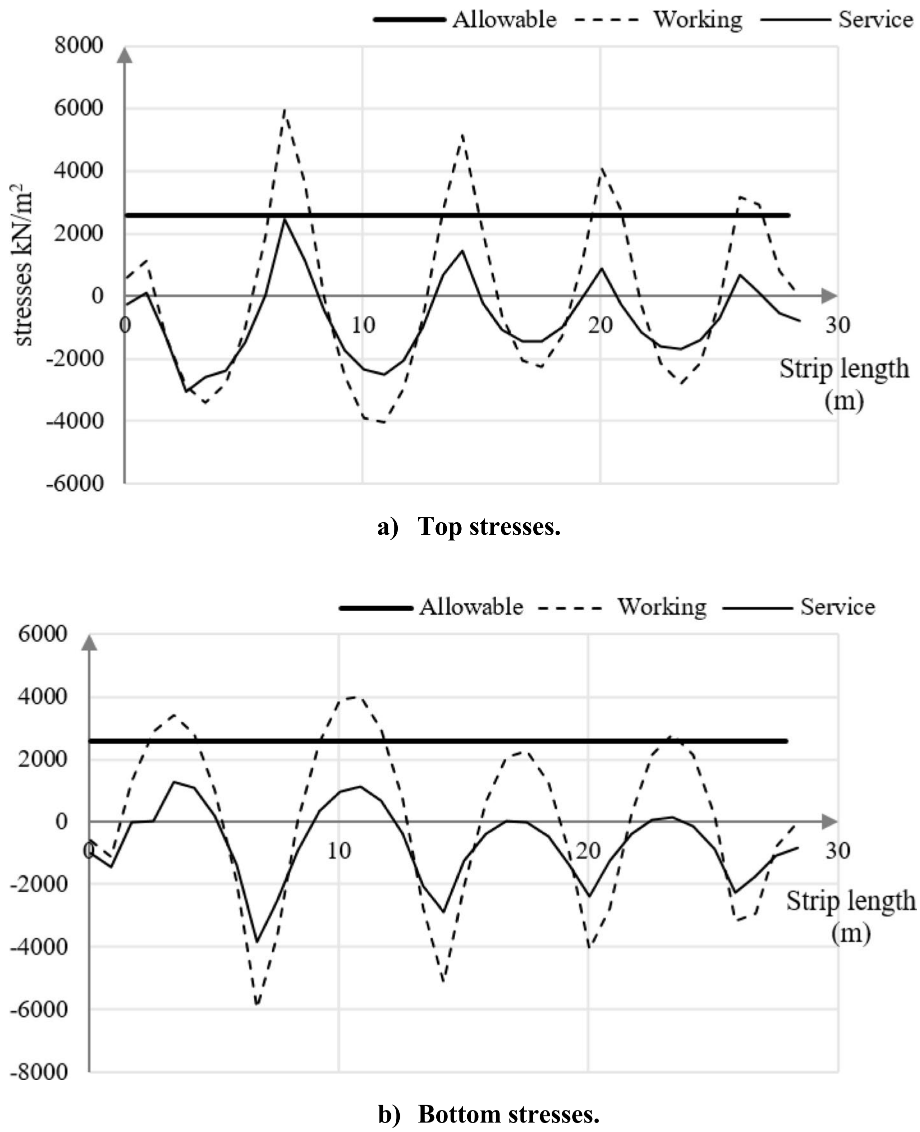


Fig. 8 Stresses along strip 10 in Example 1

the superimposed dead load and 3 kN/m^2 for the live load. This slab was previously designed via the FE method. The obtained number of strands (n_d) obtained using the traditional design is shown in Table 3, and weight of the cables was 4541.8 kg. To carry out the value engineering process, the slab is divided into 7 and 6 design strips along *X* and *Y* directions, respectively. Figure 10 shows the cable numbering and the design strips. Hence, eighty-one control points are chosen (Fig. 9). The developed tools are executed using values of n_i as given in Table 4. The same slab is reconsidered using the developed tools via the automated design (that is n_i=1). The results of the developed tools are listed in Table 4 for different n_i values. It can be seen from Table 4 that the developed tools achieve the same optimum solution regardless of the initial value of n_i whether it is n_d or 1. The cable weight of the optimized design is 3754.5 kg.

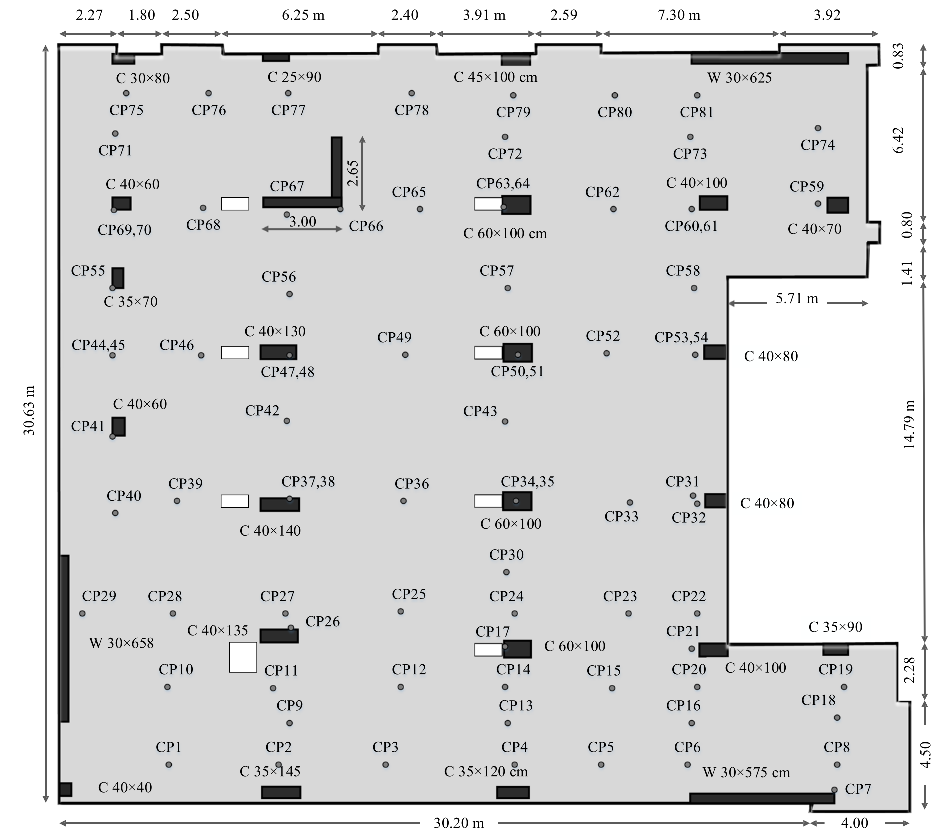


Fig. 9 Layout and dimensions and control points in Example 2

Table 3 The number of strands in each cable obtained from the traditional design method in Example 2

Cable ID	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
n_d	9	31	21	15	13	9	6	16	28	25	15	8	15

The difference between the optimized solution and that obtained from the traditional design method is $\frac{4541.8-3754.5}{4541.8} \times 100 = 14.33\%$.

Figure 11 demonstrates the final stresses acting on one proposed design strip after assigning the optimum number of strands. The final stresses are safe and optimum in all strips.

Practical example 3

The slab is shown in Fig. 12. The slab has a thickness of 0.30 m. The considered load is 2.5 kN/m^2 for the superimposed dead load and 4 kN/m^2 for the live load. This example was previously designed using the traditional design via the FE method. The total weight of the cables in the previously designed was 7821.5 kg. The number of strands for the traditional design is shown in Table 5. The purpose of this example is to demonstrate the use of the proposed tools to analyze non-rectangular geometry with circular columns and edges to carry out value engineering of already designed PT slab.

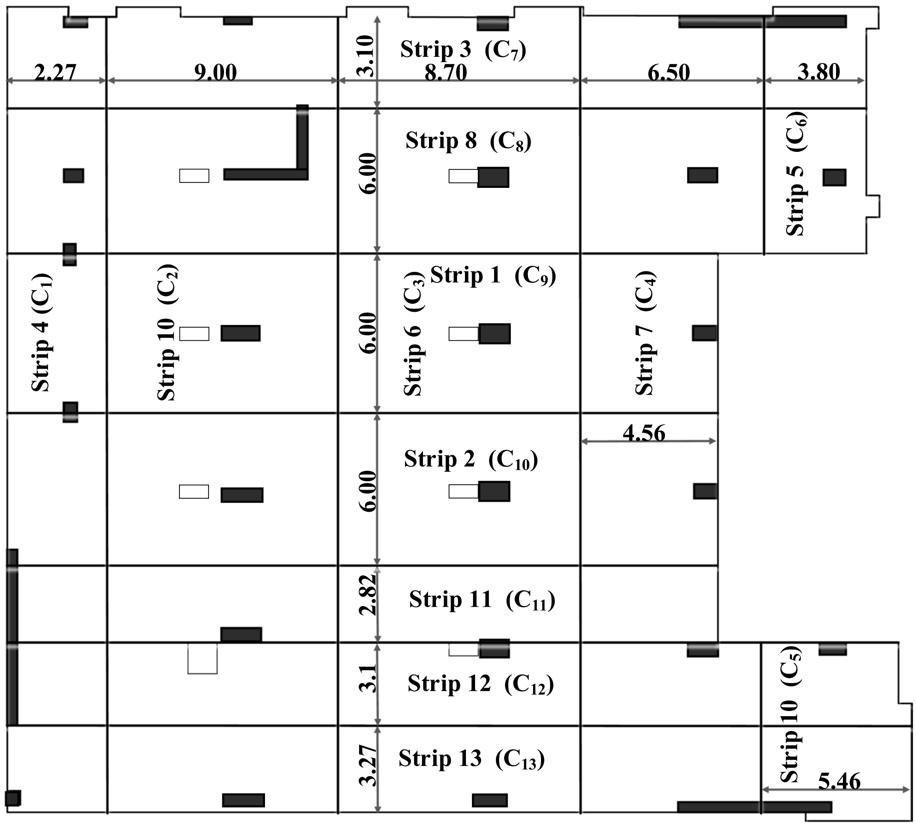


Fig. 10 Design strips and cables numbering in Example 2

Table 4 The optimum number of strands in each cable in Example 2

Cable ID	Optimization method				computed number
	Trust-constr		SLSQP		
	$n_i = n_d$	n_i=1	$n_i = n_d$	n_i =1	
C_1	4.111072	4.111072	4.09894838	4.10876191	5
C_2	8.479723	8.479722	8.46261861	8.47943075	9
C_3	8.172494	8.172494	8.14806949	8.17047863	9
C_4	8.569294	8.569294	8.54447841	8.56776934	9
C_5	4.398003	4.398002	4.39501331	4.41158023	5
C_6	3.754328	3.754328	3.75254244	3.75537962	4
C_7	9.551723	9.551723	9.50040172	9.54812062	10
C_8	18.038758	18.038758	17.98370694	18.03508169	18
C_9	27.854003	27.854003	27.77096259	27.84815877	28
C_{10}	27.540763	27.540763	27.45136146	27.53328153	28
C_{11}	19.843384	19.843384	19.74350586	19.8410958	20
C_{12}	10.287723	10.287723	10.22195745	10.28800935	11
C_{13}	14.043746	14.043746	13.93957657	14.03646345	14
Total weight					**3754.5 kg**

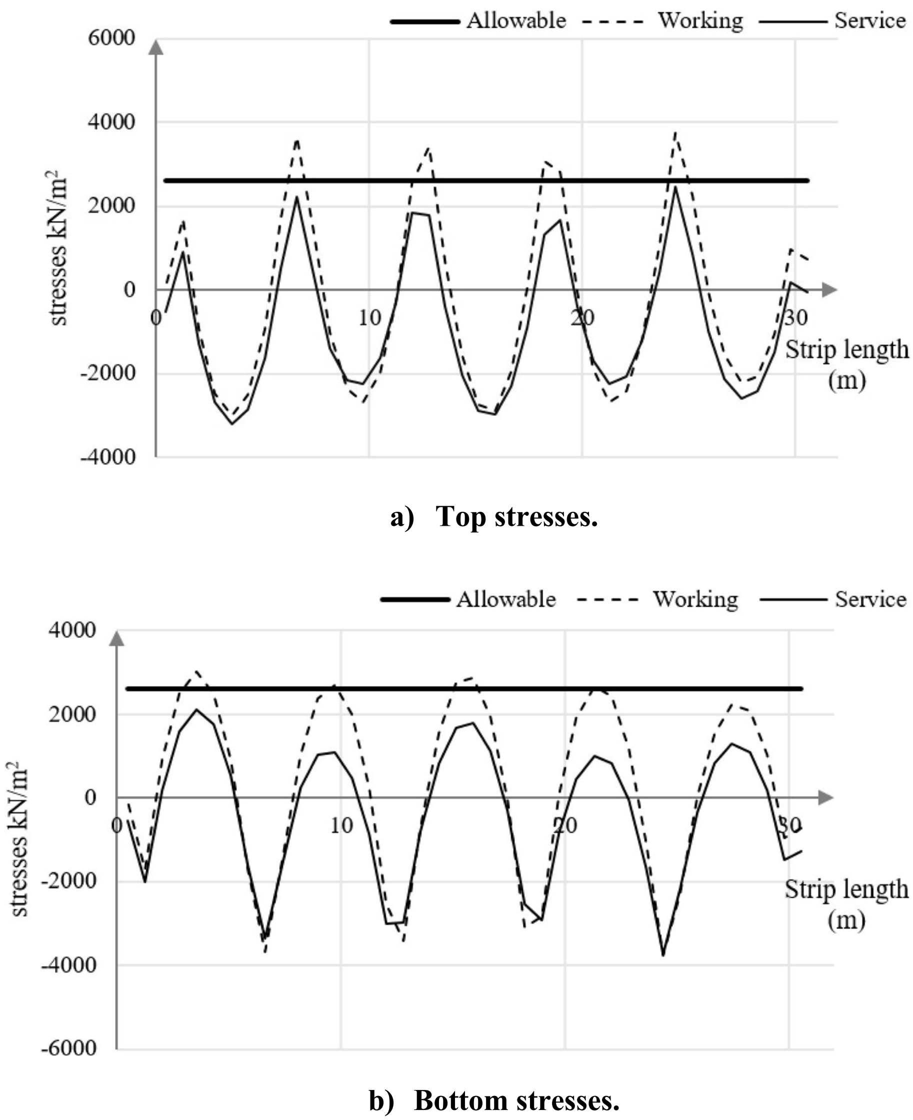


Fig. 11 Stresses along strip 6 in Example 2

In order to carry out the value engineering process, the slab is divided into 6 and 7 design strips along *X* and *Y* directions, respectively. Figure 13 shows the cable numbering and the design strips. Ninety control points are chosen (Fig. 12). This slab is considered twice via the developed tools: once using n_i=1 and second using n_i=n_d where n_d is the number of strands in each cable for the traditional design. The results are listed in Table 6. It can be seen that the optimizer always achieves the same results regardless of the initial values of n_i. The total weight of cables for the optimized design is 5537.5 kg. The difference between the optimized solution and the traditional design is $\frac{7821.5-5537.5}{7821.5} \times 100 = 29.20\%$.

Figure 14 shows the final stresses acting on one chosen design strip after assigning the optimum number of strands. The final stresses are safe and optimum in all strips. It can be seen that this example indicates high savings in material cost compared with the

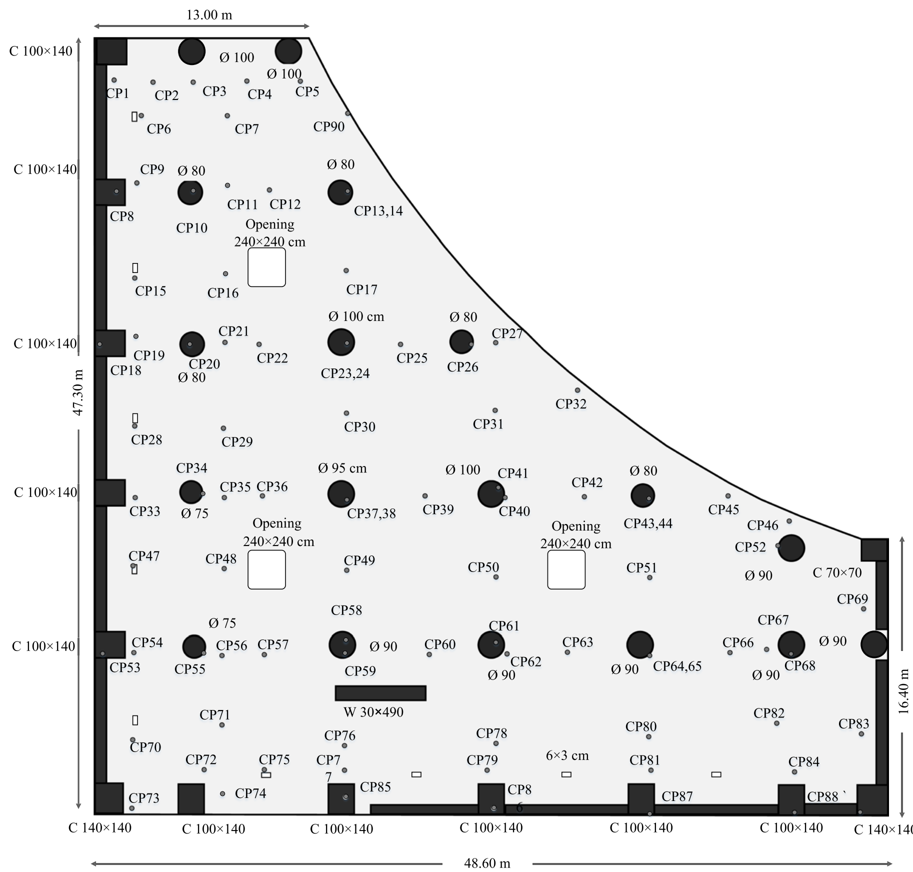


Fig. 12 Layout and dimensions of Example 3

Table 5 The number of strands for each cable in the traditional design of Example 3

Cable ID	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
n_d	9	26	36	37	36	26	10	16	34	30	41	30	15

traditional design. This might indicate that use of boundary elements in PLPAK software improves accuracy in modeling curved boundaries and circular columns [20, 21].

Conclusions

In this paper, optimization tools are developed to automate the design process of PT slabs. The same methodology could be considered an automated value engineering tool to minimize the number of strands and cables in already designed PT slab. The proposed methodology was applied to design typical slabs and can be coupled with any analysis software. Two GUIs were developed, the control points GUI, and the optimization GUI. Two algorithms of optimization were considered: the "Trust-constr" and "SLSQP" algorithm. Three practical examples were presented to demonstrate the efficiency of the developed methodology. The first practical example was presented to demonstrate the efficiency of the proposed methodology as an automated design process method. The second and third examples were presented to

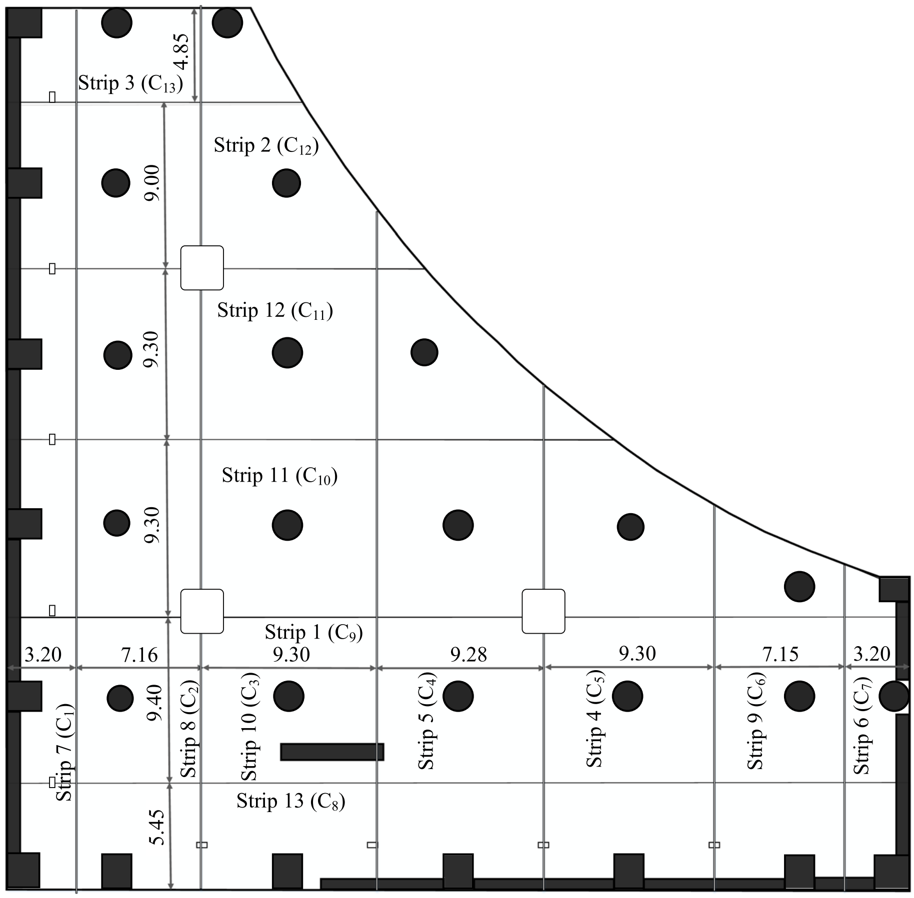


Fig. 13 Design strips along *X* and *Y* directions in Example 3

Table 6 The optimum number of strands in each cable in Example 3

Cable ID	Optimization method				Computed number
	Trust-constr		SLSQP		
	$n_i = n_d$	n_i=1	$n_i = n_d$	n_i =1	
C_1	1.107450	1.099232	1.551897	1.109556	1
C_2	16.541477	16.542106	16.476055	16.541266	17
C_3	23.564245	23.564217	23.518064	23.564194	24
C_4	20.741642	20.741719	20.735932	20.741615	21
C_5	23.195186	23.194842	23.171820	23.195145	24
C_6	15.195828	15.200255	15.184470	15.195904	16
C_7	1.253807	1.177519	1.368243	1.252234	1
C_8	4.666637	4.666631	4.717867	4.666599	5
C_9	23.841995	23.841941	23.788368	23.841959	24
C_{10}	24.807132	24.807094	24.729275	24.807086	25
C_{11}	24.244043	24.244174	24.211385	24.244006	28
C_{12}	21.656598	21.656559	21.640055	21.656576	22
C_{13}	1.000032	1.000046	1.061109	1.000003	1
Total weight					**5537.5 kg**

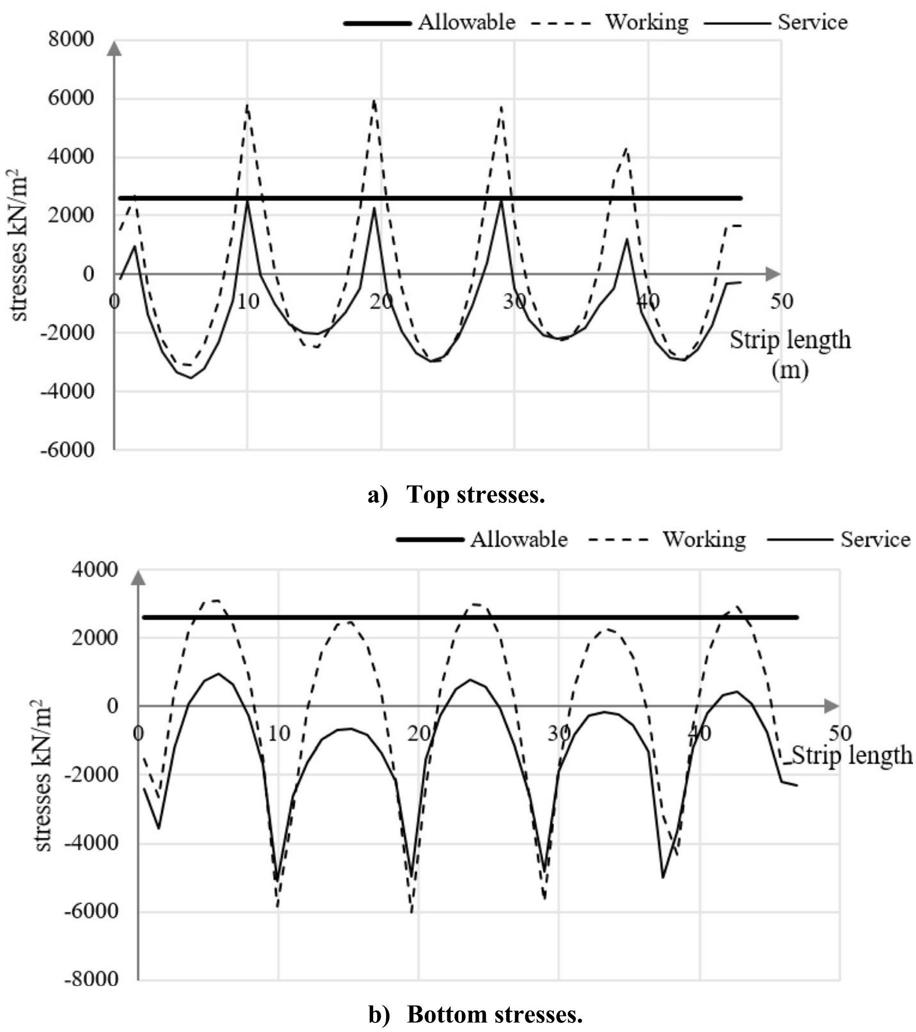


Fig. 14 Service stresses along strip 8 in Example 3

demonstrate the use of the developed technique to minimize quantities in already designed practical examples. The results indicate the following conclusions:

1. The automated design process could be used to obtain an optimized solution regardless of the experience level of the designer.
2. The developed optimizer always reaches the same optimum solution regardless of the initial number of strands.
3. The obtained savings in strand weights in the practical examples (that previously manually designed in certain design companies) are between 15 and 30 %.
4. The savings achieved might be the result of employing a boundary element simulation of the PT slab, in which is no internal discretization, and the actual geometry of the elements is modelled [22–24].

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Authors' contributions
AF wrote the manuscript, wrote the programming code, and analyzed the results. AK solved the examples, drew some figures, and prepared the practical examples. YF revised the manuscript, analyzed the results, and generated the idea of influence matrix. All authors read and approved the final manuscript.

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Availability of data and materials
Data can be shared upon request.

Declarations

Competing interests
The authors declare no competing interests.

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