Journal of Engineering and Applied Science

RESEARCH Open Access

Development of a new design methodology for slab track systems

Mohamed Hafez Aly[1], Islam Mahmoud Abou Elnaga[2], Ahmed Abdul Hay Soliman[3] and Muhammad Ahmad Diab[4*]

*Correspondence: Muhammad.diab@alexu.edu.eg

[1] Railway Engineering, Transportation Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt
[2] Roadway Engineering, Public Works Department, Faculty of Engineering, Tanta University, Tanta, Egypt
[3] Transportation Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt
[4] Structural Engineering Department, Faculty of Engineering, Alexandria University, Alexandria, Egypt

Abstract

Owing to the rapid increase in the demands of train speed and axle loads, the slab track has been introduced to replace the ballast in the classical ballasted track with reinforced concrete slab or asphalt-bearing layer to improve the track stability, strength, and durability. This paper aims to develop a new methodology for estimating the rail deformations for the most common slab track systems (BÖGL, Shinkansen, and RHEDA 2000. This methodology yielded the first design aid for slab track systems based on design equations and graphs for high-speed systems. Using a regression analysis of more than 300 finite element models which are validated by experimental tests, the relationship between the rail deflection, modulus of elasticity for subgrade and replacement, and the replacement thickness was determined for the most common slab tracks under the American (AREMA) and European (EN) loads. According to EN, it was found that the minimum modulus of elasticity for subgrade to fulfill the rail deflection criterion without a replacement soil ranges from 128 to 143 MPa for the most common slab track systems; meanwhile, for AREMA, it ranges from 59 to 70 MPa. Furthermore, for these slab track systems, one simple design chart was introduced to aid engineers with the design of the slab track replacement layer according to each design code.

Keywords: Railway slab track, BÖGL slab track, Shinkansen slab track, RHEDA 2000 slab track, Nonlinear analysis, Parametric study, AREMA and EN specifications, Midas GTS NX software

Introduction

The ballastless track was used in railway lines for the first time since 1970s in Germany [1]. It was utilized to overcome the issues triggered by the ballasted track such as ballast churning up at high speeds and deterioration of the ballast over time in addition to cope up with the high axle loads of the freight trains [1, 2]. The slab track has exchanged the ballast material layer in the traditional ballasted track with either reinforced concrete slab (precast or casted in situ) or asphalt layer which increases the track stability, stiffness, and durability over the time [3, 4].

The ballastless track consists of two parts, namely the superstructure and the substructure. The superstructure elements of the non-ballasted track are steel rails, reinforced concrete sleepers (or concrete blocks), fastening system to attach rail into position, concrete-bearing layer (CBL) or asphalt-bearing layer (ABL), and hydraulic-bonded layer (HBL) which is a mixture of aggregates combined by a hydraulic binder (such as cement); in some slab track types, a layer of cemented-asphalt mixture (CAM) is injected between CBL and HBL to provide flat surface and connection between CBL and HBL [3]. On the other hand, the substructure components of the ballastless track are frost protective layer (FPL) which is a mix of sands and gravel to prevent the frost heave from affecting CBL, replacement layer (in case of poor-quality subgrade), and subgrade soil which is natural ground soil at site [2, 5–7]. Figure 1 illustrates the main elements of the slab track system.

Studying railway systems, especially slab track, by manual calculations is daunting and time-consuming. Therefore, the employment of finite element software is considered to save efforts. Some researchers have studied the slab track using the finite element method or experimental physical tests to investigate its structural behavior and compare it to the classical ballasted track. These studies have included static, dynamic, and cyclic axle loads for different slab track systems on grade, tunnels, and bridges [9–11].

G. Michas built a finite element model with ABAQUS software to investigate the static linear behavior of RHEDA 2000 slab track using EN loading model. Michas has found that the slab track deformation is smoother than ballasted track because of distributing the loads in much larger areas. On the other side, the ballasted track deformation is sharper near the points where the loads are located and almost undeformed at the areas far from the loads [4]. Slab track has shown less rail top level displacement than ballasted track by approximately 60%. *T. Čebašek et al.* tested the slab track (BÖGL) and ballasted track to investigate their behavior under static and cyclic loading using GRAFT apparatus in Heriot-Watt University. *Čebašek* has figured out that the concrete slab track performed significantly better in terms of cumulative settlement and rail deformation when compared to the ballasted track (80% reduction due to slab track). *Čebašek* found out that the ballasted track produces higher displacement measured at rail top level than BÖGL slab track. The major reason for the observed higher displacement of the ballasted track was caused by the unbound nature of ballast [12]. *T. Wang et al.* obtained a practical methodology to design the slab track substructure for high speed (400 km/h) by reducing the long-term deformations in the soil using dynamic analysis. They have validated their work through field observations. Their model is function of modulus of subgrade reaction and plasticity index of subgrade soil, and it can be utilized for earthquake cases and high-speed systems [9]. *A. Ramos et al.* have performed numerical models to calibrate with the experimental physical tests held by *T. Čebašek et al.* in

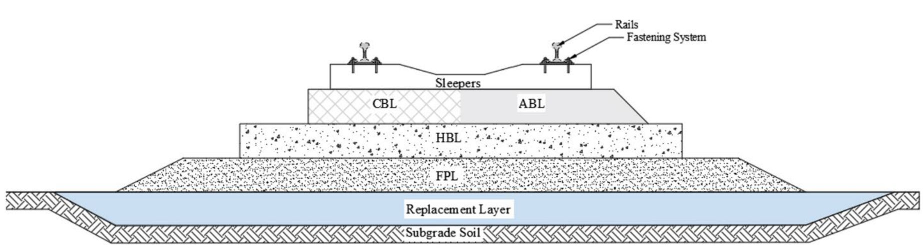

Fig. 1 Components of the slab track system [3, 4, 8]

Heriot University laboratory. These numerical models have shown great match with the experimental tests of *T. Čebašek*. The main aim of their work was to develop an empirical model for predicting long-term track deformation for ballasted and slab track [13].

D. Thölken et al. carried out two experimental tests on slab track (BÖGL) using GRAFT II apparatus in Heriot University to study its structural behavior under static and dynamic loads with different scenarios. In addition, numerical models have been built with ABAQUS to be calibrated with these experimental results, and these models showed excellent agreement with the physical test [14]. They found that modulus of elasticity is the most crucial factor for subgrade soil to be put into consideration while investigating slab track systems. *X. Cui et al.* studied the differential settlement in slab track systems (CRTS II), and they used finite element method by creating a 3D model based on damage mechanics theories. Cui found that with the increasing of settlement value, the concrete base is affected first. If the settlement value still grows, the precast slab is damaged. Eventually, damage to the prefabricated RC slab and concrete base ruins the integrity of the longitudinal connection system of the CRTS II slab track [15]. *M. Atalan et al.* investigated the behavior of slab track with asphalt-bearing layer using analytical and numerical techniques to study the effect of dynamic forces on high-speed railway lines (HSL). They have found that the use of asphalt-bearing layer in railway slab tracks has beneficial impacts, such as increasing the bearing capacity of the soil and increasing the vertical stiffness of the track [10]. *Aly et al.* studied the most common slab track systems (RHEDA 2000, BOGL, Shinkansen) under different loading models (EN and AREMA) to estimate the most efficient slab track system. They created a nonlinear finite element model using Midas GTS NX and validated through experimental work [3]. *Aly et al.* discussed the difference between the linear and nonlinear behavior for slab track systems [16]. They figured out that the superstructure of the slab track should be modelled as linear materials following the Hook's law model. On the other hand, the substructure of the slab track should be treated as nonlinear materials following Mohr-Coulomb model [17]. *Aly et al.* also compared the nonlinear behavior of the standard section for the most common slab track systems under EN and AREMA to determine the most structurally efficient slab track. They have found that RHEDA 2000 produces the least straining actions and stresses on soil layers which means it will maintain its geometric section over time better than BOGL and Shinkansen. They also have found that rail displacement is the most governing factor in the design of slab track systems [16].

Esen et al. have created slab track and ballasted track finite element models to replicate the full-scale physical models created in the laboratory using GRAFT II apparatus in Heriot-Watt University. They have calibrated the numerical models with the experimental models [18]. They have studied the dynamic loads (linearly) on the slab track and ballasted track and found that at high speed, the slab track showed less displacement than the ballasted track due to the existence of the hydraulic bonded layer and concrete bonded layer. On the other hand, at low speeds, they both showed the same behavior (roughly) regarding the rail displacement for several train loads [18].

From the aforementioned studies, the lack of a direct methodology that can be utilized as design aid using EN and AREMA loads in case of multiple soil layers for slab track systems for civil engineers has motivated this research study. This paper aims to develop relationships between rail displacement, modulus of elasticity for subgrade and replacement, and the replacement thickness for specific slab track systems, namely BÖGL, Shinkansen, and RHEDA 2000. These relationships can be considered as the 1st design aid for slab track systems based on design equations and graphs. For practical engineers, it is much easier to use one graph to obtain their design/analysis rather than using several long equations. Thus, the proposed design aid was presented not only as design equations but also as one design graph for each studied slab track system.

Research methodology

The design of slab track systems should include several factors such as rail displacement, slab bending moments, replacement stresses, and subgrade stresses. One of the most vital criteria to be taken into consideration is rail displacement. Thus, a new methodology to design the slab track has been built using Midas GTS NX software and employing around 300 finite element models (validated by experimental testing held in Heriot-Watt University). A detailed information regarding the used finite element models and its validation can be found in a previous research conducted by the authors [16]. These finite element models have been built using various types of soils (having different modulus of elasticity) to produce relationship equations and graphs that can be utilized for slab track design purposes (using a regression analysis).

The loading patterns and code specifications utilized in this paper are the European norms (EN), and American Railway Engineering and Maintenance of The Way (AREMA). The type of load employed in this paper regarding EN specifications is load model "LM 71" because it represents the static vertical effect of normal service traffic loads, LM 71 consists of 4 axles loads of 250 kN, and a continuous load of 80 kN/m distributed on the track lengthwise as shown in Fig. 2 [19].

The EN specifications suggests the use of Eisenmann dynamic impact factor "I" which is taken according to equation (Eq. 1) [6, 20].

$$\mathbf{I} = 1 + \phi \times t \times s \tag{1}$$

The parameter "t" can be taken as 0.1, 0.2, or 0.3 depending on track quality (as good, moderate, or bad condition track), "s" can be assumed 1, 2, or 3 based on the selected confidence level for obtaining the maximum rail deflection (84.1%, 97.7%, and 99.9%), and "ϕ" can be obtained from the following equation (Eq. 2):

$$\phi = 1 + \frac{V - 60}{380}, \mathit{for}\ 200 < \mathit{speed} \leq 300\, km/hr \tag{2}$$

where V is the speed in km/h, for speed of 300 km/h, assuming track quality is good ($t = 0.1$), and confidence level is 99.9% ($s = 3$); the Eisenmann impact factor "I" according to EN equal to 1.50.

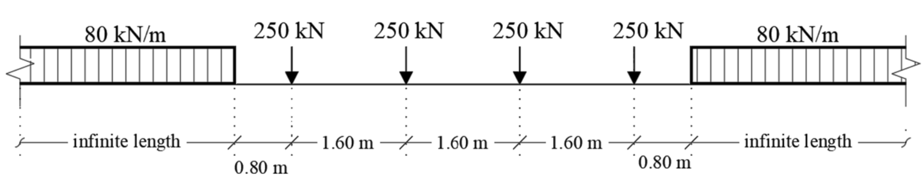

Fig. 2 Load model LM71-UIC(71)- [19]

Eccentricity of vertical loads according to EN must be taken as 11% of train loads on one rail and −11% on the other rail. Another factor "α" is employed to adjust the loading model to match the lighter or heavier traffic loads for EN only [19]. Factor "α" is mostly taken as 1.33 across European nations. In addition, European slab track design must be within the allowable rail deflection of 2 mm [5].

Cooper E80 is the type of loading configuration suggested by AREMA, and the "80" value stands for the heaviest axle load in this load model in kilo pounds [21, 22]. Cooper E80 consists of two steamed locomotives with four axles of 355.8 kN, two leading axles of 177.9 kN, and two tender wagons comprised of four axles of 231.3 kN. In addition, the trailing linearly distributed load is about 116.8 kN/m as depicted in Fig. 3 [22].

AREMA recommends the use of impact factor according to equation (Eq. 3) [20].

$$\boldsymbol{I} = 1 + \frac{D_{33} \times V}{D_{wheel} \times 100} \tag{3}$$

where V is speed in mph, D_{33} is the standard diameter of a 33 inches wheel, and D_{wheel} is the actual train wheel diameter in inches. For speed of 300 km/h (186.41 mph), $D_{33} = 33$ inch, and $D_{wheel} = 36$ inch, and the impact factor "I" according to AREMA = 2.70. The eccentricity of vertical loads according to AREMA must be taken as 20% of train loads on one rail and −20% on the other rail. In addition, AREMA slab track design must be within the allowed rail vertical displacement which is 6.35 mm [22].

Finite element model building

The used finite element model (FEM) developed by the authors [16] used Midas GTS NX software utilizing the nonlinear static analysis [3, 16, 17]. The nonlinear static analysis parameters are as follows: max iterations = 50, increments = 20%, and the allowable tolerance is 0.001 in displacement and energy.

The geometric models for each slab track type (BÖGL, Shinkansen, and RHEDA 2000) have been carried out according to their standard section. The thickness of the replacement layer has changed from 0 to 2.50 m because EN specifications must assure that 2.50 m of soil under the slab track is studied [5]. Furthermore, the side extensions of the replacement layer and subgrade soil have been selected as 3.0 m to assure well-distributed stress in soil layers. The materials mechanical properties for the slab track models have been assumed as per Table 1 according to the experimental tests by D. Thölken et al. [13, 14]. Regarding cohesionless soils, the cohesion "C" is taken as small value to avoid misleading outputs according to Midas Manu [23]. Hybrid mesher has been utilized in this work because it is preference when the model is simple as it guarantees more accuracy in stress calculations [23]. A previous mesh study has been performed by the authors [16], and it was found that there is no need to use mesh size less than 10 cm since the differences in the rail displacement are insignificant while computational costs can be saved. The boundary conditions (supports) for the slab track models have been selected to represent the surrounding field

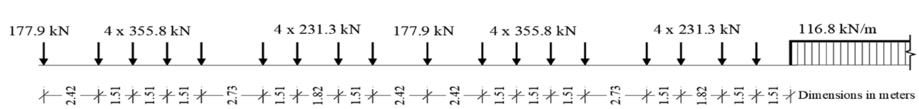

Fig. 3 Cooper E80 load model [21, 22]

Table 1 The mechanical properties of the materials, modelling techniques, and mesh sizes used for the slab track FEMs

Component	Material description	Modelling technique	Mesh size (m)	γ (kN/m³)	E (MPa)	υ	Φ	C (KPa)
UIC 60 rail	Steel	Linear-elastic model	0.10	78.5	210,000	0.30	-	-
CBL	Reinforced concrete		0.20	25.0	36,000 34,000[a]	0.20	-	-
Sleepers	Reinforced concrete		0.20	25.0	34,000	0.20	-	-
CAM (grouting)	Cemented-asphalt mixture		0.30	23.0	22,500	0.25	-	-
HBL	Mixture of aggregates with hydraulic binder		0.30	24.0	17,870	0.20	-	-
FPL	Mixture of sands and gravel	Mohr-Coulomb model	0.50	21.44	120	0.30	35	1
Replacement soil	Well-graded limestone (AASHTO: A-1-b)		0.50	20.91	100 to 400	0.30	35	1
Subgrade soil	Sandy silt (AASHTO: A-2-6)		0.50	17.00	20 to 100	0.30	10	10

[a] Value concerns the RHEDA 2000 slab track

soil. The transition at X and Y direction for both left/right and front/back is constrained; meanwhile, the transitions at X, Y, and Z are constrained for the bottom surface of the models.

Methodology validation

The validation for this methodology has been performed by comparing the results of the finite element model with an experimental test held in Heriot-Watt University by D. Thöklen et al. This experiment has utilized the slab track of BÖGL under specific loading procedures using GRAFT-II apparatus (GRAFT stands for Geo-pavement and Railways Accelerated Fatigue Testing II) as shown in Fig. 4. The main dimensions of the used slab track in this experimental test are shown in Fig. 5.

D. Thölken et al. have tested the BÖGL slab track using three different scenarios (S_1, S_2, and S_3) with the same mechanical properties as per Table 1 (modulus of elasticity for both subgrade and replacement is taken 400 MPa). In the 1st scenario, an axle load of 130 kN has been used divided onto three adjacent sleepers as 25%, 50%, and 25%. The 2nd scenario is to employ an axle load of 170 kN distributed on three sleepers as 25%, 50%, and 25%. The 3rd scenario is to apply an axle load of 258 kN with equal percentages for each sleeper (33.33% for each sleeper); these loading scenarios are illustrated in Fig. 6.

Figure 7 illustrates the comparison between the displacement obtained from the Midas GTS NX model and the experimental test measured at the actuators (on rails) and at LVDTs (on sleepers, LVDT stands for linear variable differential transformer). The average differences between the finite element models and experimental tests are less than 6%. This means that the finite element model built by Midas GTS NX shows great match with experimental work.

Fig. 4 GRAFT II apparatus used in the D. Thöklen experimental test [13, 14]

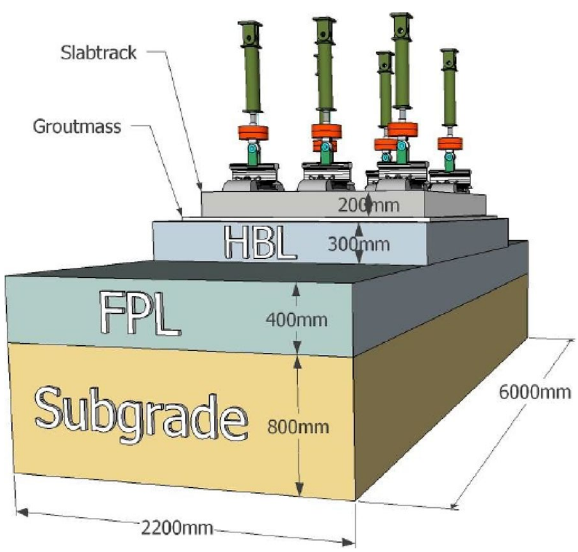

Fig. 5 The dimensions of utilized slab track system in D. Thölken experimental test [12, 14]

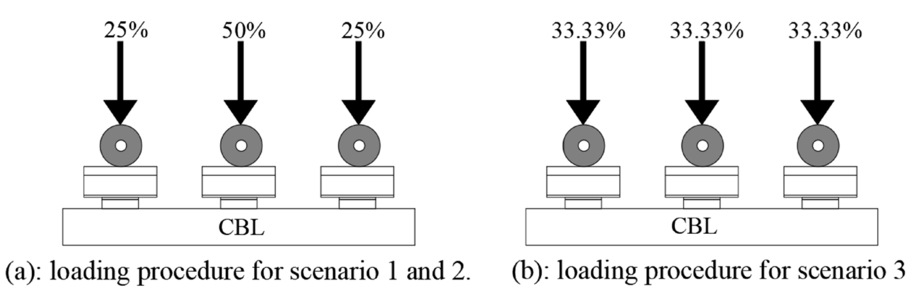

Fig. 6 Loading procedure for the three scenarios used in D. Thölken experimental test [14]. **a** Loading procedure for scenarios 1 and 2. **b** Loading procedure for scenario 3

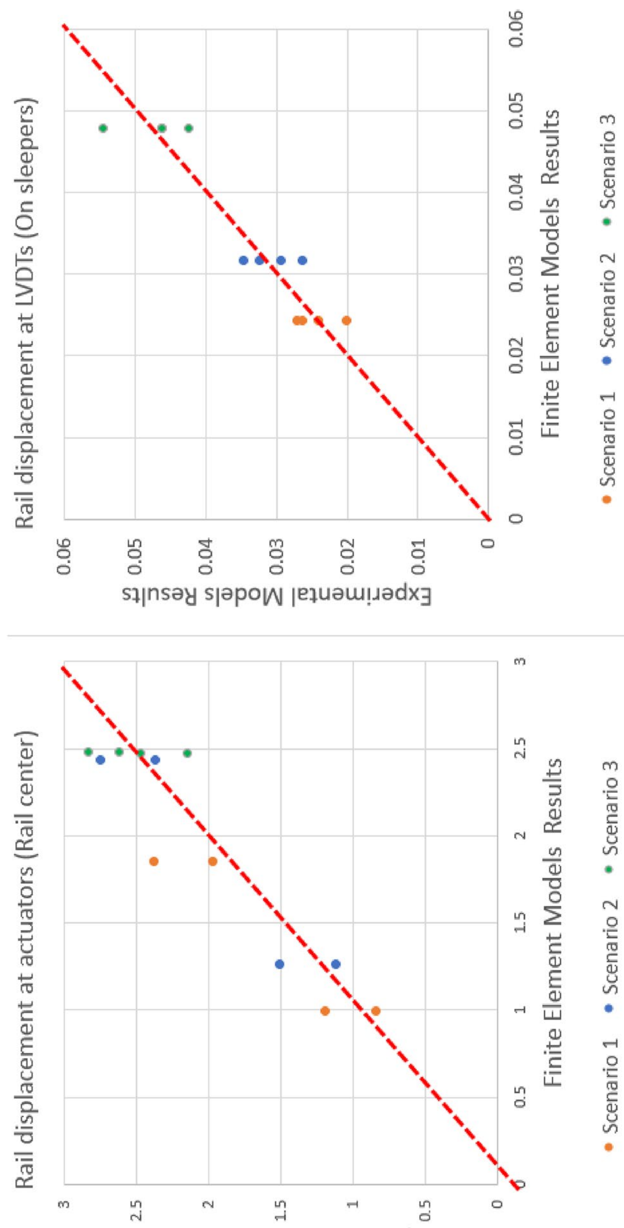

Fig. 7 Comparison between D. Thölken experimental model and Midas GTS NX model

Results and discussion

The new methodology has utilized two parametric studies to obtain the design equations and charts to determine the rail displacement for the most common slab track systems (BÖGL, Shinkansen, and RHEDA 2000), and these parametric studies have employed both EN and AREMA specifications. The conducted equations and charts can be considered the 1st design aid for civil engineers to design new or analyze existent slab tracks.

EN parametric study

In EN parametric study, the elasticity modulus of subgrade (E_{sub}) has been changed from 60 to 120 MPa because the minimum allowable E of a subgrade soil is 60 MPa according to EN specifications [5, 6], and the soil having E of 120 MPa does not require any replacement (as concluded from this parametric study). Meanwhile, the elasticity modulus of replacement soil (E_{rep}) has been changed from 150 to 400 MPa and depth of replacement soil from 0 to 2.5 m as the specifications require the subgrade soil to be suitable up to a depth of 2.5 m [2, 5].

Using regression analysis, the EN parametric study on the common three slab track types has been carried out, and this study has employed around 150 finite element models for these three slab types through Midas GTS NX software. The general equation (Eq. 4) has been developed by linear regression to describe the relationship between the rail deflection (y in mm), the elasticity modulus of subgrade soil (E_{sub} in MPa), the elasticity modulus of replacement soil (E_{rep} in MPa) and the thickness of the replacement soil (d in meters):

$$y = y_0 - m \times d \tag{4}$$

$$\mathrm{y_o} = \beta \times \mathrm{E}_{\mathrm{sub}}^{-\alpha} \tag{5}$$

$$m = -a_1 \times 10^{-6} \mathrm{E}_{rep}^2 - a_2 \times 10^{-3} \mathrm{E}_{rep} + a_3 \times 10^{-4} \mathrm{E}_{sub}^2 - a_4 \times \mathrm{E}_{sub} + a_5 \tag{6}$$

The term "y_o" is the deflection (mm) in subgrade soil only (E_{sub}), and the term "m" represents the subgrade and replacement soil characteristics as it depends on E_{sub} and E_{rep}. Parameters "α" and "β" have been determined for each *slab track type* by power regression analysis, while parameters "a_1" to "a_5" have been evaluated according to each *slab track type* through two-step polynomial regression analysis. These equations (Eqs. 4, 5 and 6) have a regression factor R^2 equal to 0.98.

EN parametric study application on BÖGL

The EN parametric study on BÖGL has utilized the loading criteria (Section 1: Load model LM70), methodology (Section 2), finite element model and analysis (Section 3), and the standard section of BÖGL as depicted in Fig. 8. The regression analysis for the numerical models of BÖGL has obtained the terms "y_o" and "m" as follows:

$$\mathrm{y_o} = 128.97 \times \mathrm{E}_{\mathrm{sub}}^{-0.86} \tag{5-1}$$

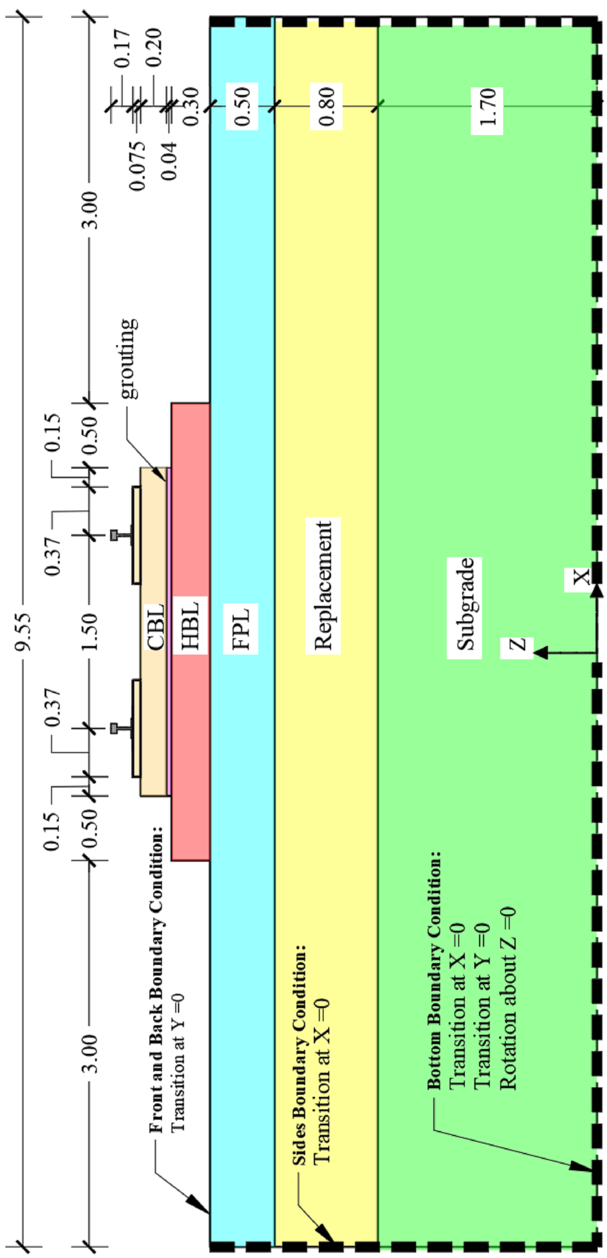

Fig. 8 BÖGL slab track standard dimensions

$$m = -4.95\times10^{-6}E_{rep}^2 - 4.1\times10^{-3}E_{rep} + 1.23\times10^{-4}E_{sub}^2 - 0.0335E_{sub} + 1.89 \quad (6\text{-}1)$$

The relationship between the rail displacement and modulus of elasticity for subgrade soil can be represented in Fig. 9, and this figure has been created through power regression analysis using (Excel and SPSS). Figure 9 can be used to determine the minimum E_{sub} that does not require replacement; in this case, the minimum $E_{sub} = 130$ MPa (the point of intersection between y_o and the dotted line which represents the EN allowable limit of deflection "2.0 mm").

For practical engineers, it is much easier to use one graph to obtain their design/analysis rather than using several long equations. Thus, the general equation for BÖGL slab track (EN) is represented graphically in Fig. 10. This figure indicates the relationship between the vertical rail displacement in mm (*y*-axis), the elasticity modulus for subgrade soil in MPa (E_{sub} from 60 to 120 MPa) and elasticity modulus for replacement layer soil in MPa (E_{rep} from E150 to E400 MPa), and the thickness of replacement soils in meters (*x*-axis). The values between the lines shown in Fig. 10 can be integrated.

EN parametric study application on Shinkansen

The EN parametric study on Shinkansen has utilized the same parametric factors (EN loading pattern, finite element analysis type, and the boundary condition) as per BÖGL slab track and the standard section of Shinkansen as illustrated in Fig. 11. The numerical models for Shinkansen slab track system have resulted in the terms "y_o" and "m" by regression analysis as follows:

$$y_o = 117.93 \times E_{sub}^{-0.821} \quad (5\text{-}2)$$

$$m = -5.38\times10^{-6}\times E_{rep}^2 + 4.4\times10^{-3}\times E_{rep} + 2.45\times10^{-4}\times E_{sub}^2 - 5.58\times10^{-2}\times E_{sub} + 2.78 \quad (6\text{-}2)$$

The relationship between the rail displacement and modulus of elasticity for subgrade soil can be represented in Fig. 12. Figure 12 can be used to determine the minimum E_{sub} that does not require replacement; in this case, the minimum $E_{sub} = 143$ MPa.

The general equation for Shinkansen slab track (EN) is represented graphically in Fig. 13. This figure indicates the relationship between the vertical rail displacement in mm (*y*-axis), the elasticity modulus for subgrade soil in MPa (E_{sub} from 60 to 120 MPa)

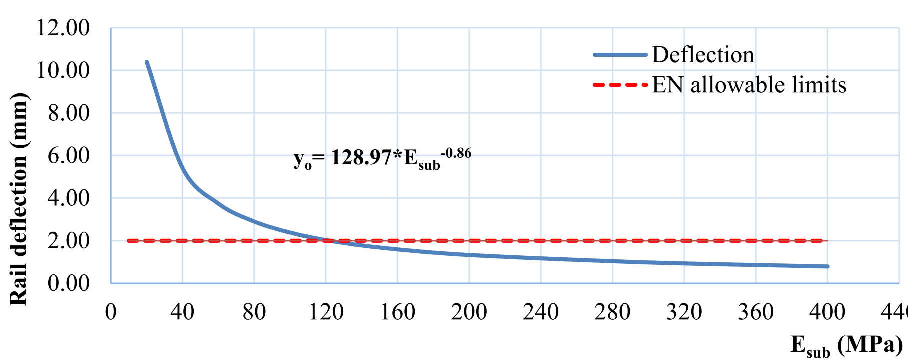

Fig. 9 Relationship between rail deflection (y_o) and subgrade elasticity modulus (E_{sub}) for BÖGL (EN)

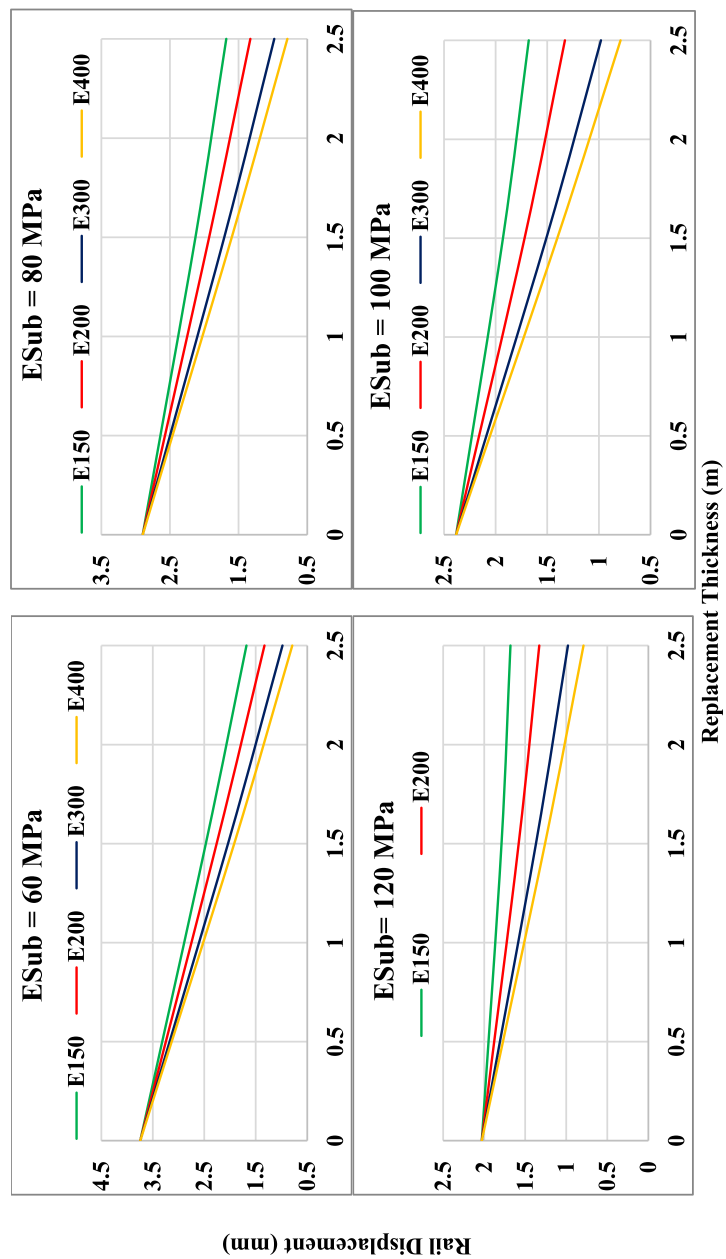

Fig. 10 Relationship between elasticity modulus of replacement soil and maximum rail deflection (*y*-axis) for different subgrade soils (different E_{sub}) and thicknesses of replacement soil layer (*x*-axis) for BÖGL slab track (EN)

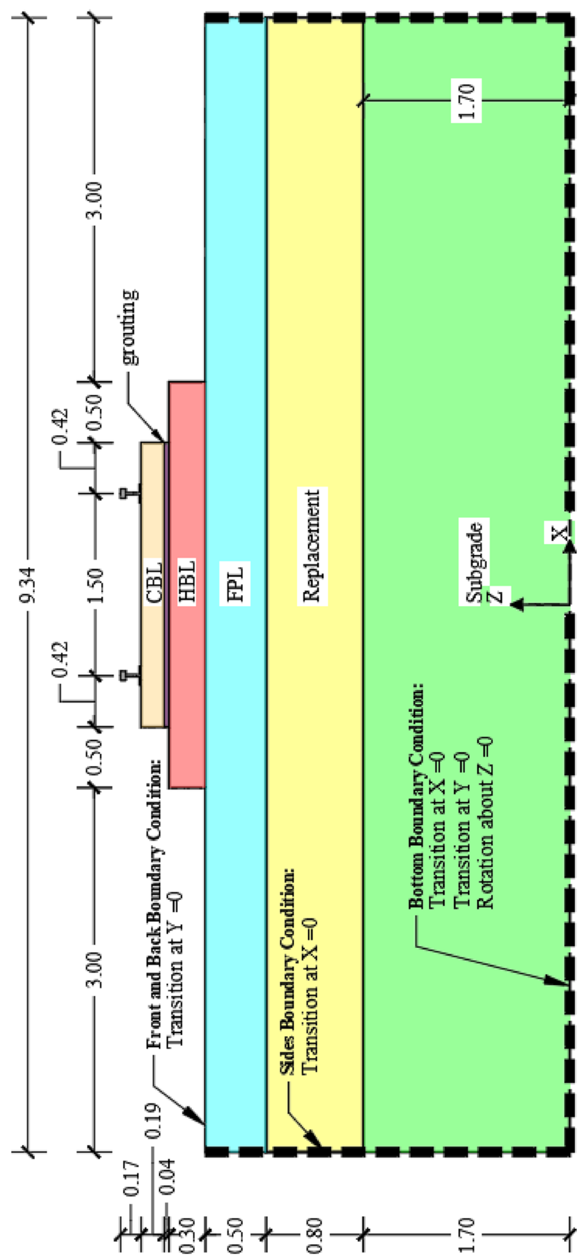

Fig. 11 Shinkansen slab track standard dimensions

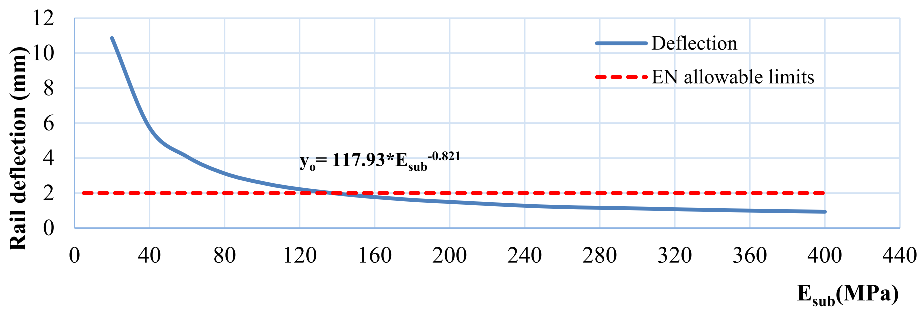

Fig. 12 Relationship between rail deflection (y_o) and subgrade elasticity modulus (E_{sub}) for Shinkansen (EN)

and elasticity modulus for replacement layer soil in MPa (E_{rep} from E150 to E400 MPa), and the thickness of replacement soils in meters (x-axis). The values between the curved lines shown in Fig. 13 can be simply interpolated.

EN parametric study application on RHEDA 2000

The EN parametric study on RHEDA 2000 has utilized the same parameters as BÖGL slab track, and the standard section of RHEDA 2000 as depicted in Fig. 14. The numerical models for RHEDA 2000 slab track type have found the terms "y_o" and "m" by regression analysis as follows:

$$y_o = 114.84 \times E_{sub}^{-0.836} \tag{5-3}$$

$$m = -5.0\times10^{-6}\times E_{rep}^2+4.07\times10^{-3}\times E_{rep}+1.22\times10^{-4}\times E_{sub}^2-3.28\times10^{-2}\times E_{sub}+1.84 \tag{6-3}$$

The relationship between the rail displacement and modulus of elasticity for subgrade soil can be represented in Fig. 11. From Fig. 11, the minimum E_{sub} that can be used without required replacement soil equals $E_{sub}=128$ MPa (Fig. 15).

The general equation for RHEDA 2000 slab track (EN) is represented graphically in Fig. 16. This figure indicates the relationship between the vertical rail displacement in mm (y-axis), the elasticity modulus for subgrade soil in MPa (E_{sub} from 60 to 120 MPa) and elasticity modulus for replacement layer soil in MPa (E_{rep} from E150 to E400 MPa), and the thickness of replacement soils in meters (x-axis). The values between the curved lines shown in Fig. 16 can be simply interpolated.

AREMA parametric study

Regarding the AREMA parametric study, the elasticity modulus of subgrade (E_{sub}) has been changed from 20 to 50 MPa. Additionally, the elasticity modulus of replacement (E_{rep}) has been changed from 100 to 400 MPa and thickness of replacement from 0 to 2.5 m. Using regression analysis, AREMA parametric study has been carried out regarding the most common slab track systems (BÖGL, Shinkansen, and RHEDA 2000) with their standard section. This study has employed approximately 150 finite element models (FEMs) for the three slab track systems to obtain the relationship between the rail displacement, modulus of elasticity for subgrade, and replacement soil as well as the replacement soil thickness. The general equation for AREMA is typically the same as equation (Eq. 4 in EN); meanwhile, the

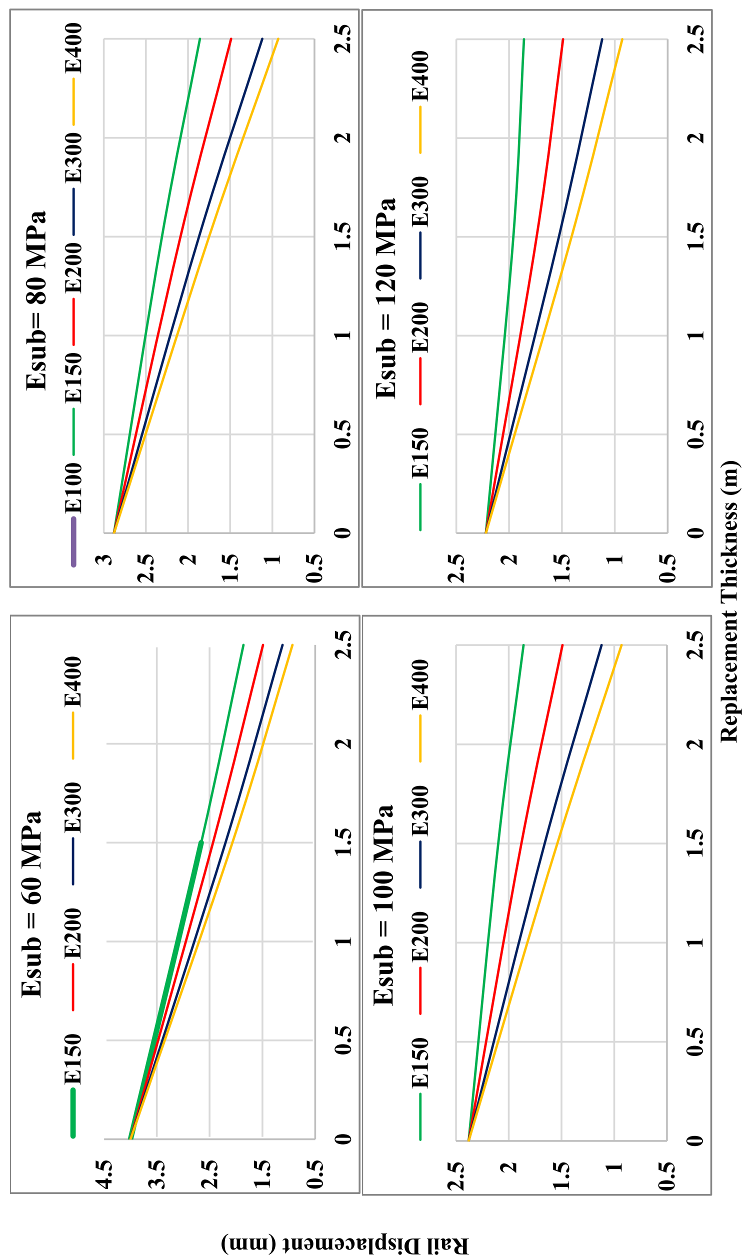

Fig. 13 Relationship between elasticity modulus of replacement soil and maximum rail deflection for different subgrade soils (different E_{sub}) and thicknesses of replacement soil layer for Shinkansen slab track (EN)

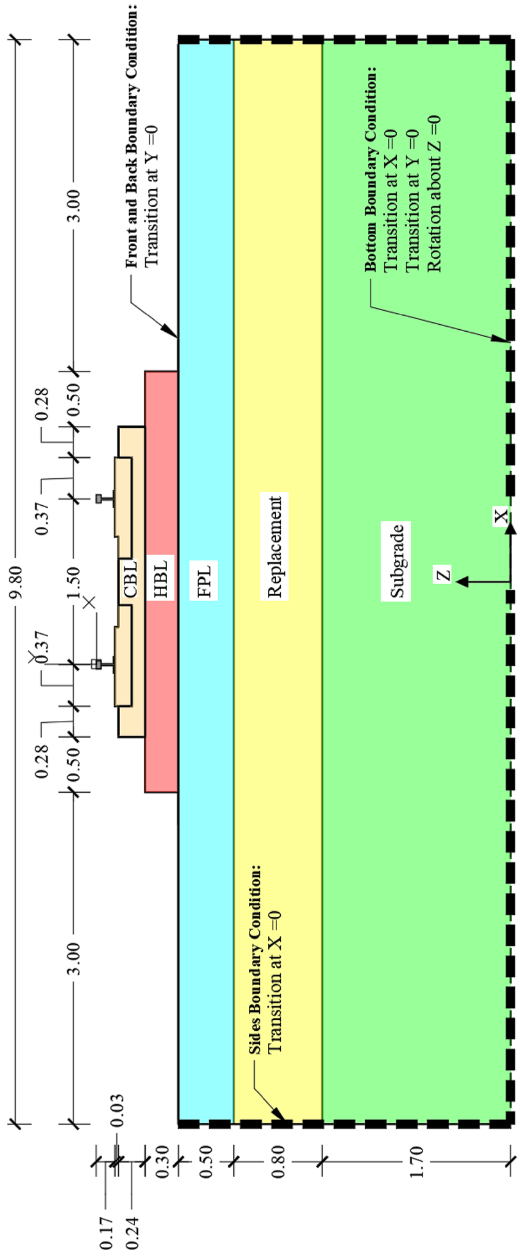

Fig. 14 The standard section of RHEDA 2000 slab track

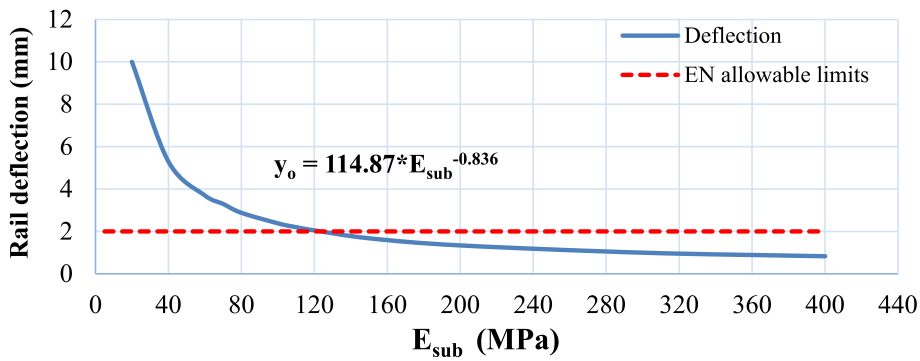

Fig. 15 Relationship between rail deflection (y_o) and subgrade elasticity modulus (E_{sub}) for RHEDA 2000 (EN)

term "m" has been derived as per Eq. 8 through 3rd-degree polynomial regression instead of 2nd-degree as per EN:

$$y = y_0 - m \times d \tag{4}$$

$$y_o = \beta \times E_{sub}^{-\alpha} \tag{7}$$

$$m = a_1 \times 10^{-7} E_{rep}^3 - a_2 \times 10^{-4} E_{rep}^2 - a_3 \times 10^{-3} E_{rep} + a_4 \times 10^{-4} E_{sub}^2 - a_5 \times E_{sub} + a_6 \tag{8}$$

where the term "y_o" is the deflection (mm) in subgrade soil only, d is the thickness of the replacement layer in meters, and the term "m" represents the subgrade and replacement soil characteristics as it depends on E_{sub} and E_{rep} in MPa. Parameters "α" and "β" have been determined according to slab track type by power regression analysis, while parameters "a_1" to "a_6" have been evaluated according to slab track type through two-step polynomial regression analysis. These equations have been derived with a regression factor $R^2 = 0.987$. The terms "y_o" and "m" have been obtained by regression analysis for the studies three types of slab track systems as follows:

For BÖGL slab track system:

$$y_o = 200.82 \times E_{sub}^{-0.839} \tag{9}$$

$$m = 1.55 \times 10^{-7} \times E_{rep}^3 - 1.32 \times 10^{-4} \times E_{rep}^2 + 3.7 \times 10^{-2} \times E_{rep} + 3.93 \times 10^{-3} \times E_{sub}^2 - 0.404 \times E_{sub} + 9.21 \tag{10}$$

For Shinkansen slab track system:

$$y_o = 161.066 \times E_{sub}^{-0.754} \tag{11}$$

$$m = 1.37 \times 10^{-7} \times E_{rep}^3 - 1.2 \times 10^{-4} \times E_{rep}^2 + 3.54 \times 10^{-2} \times E_{rep} + 3.92 \times 10^{-3} \times E_{sub}^2 - 0.408 \times E_{sub} + 9.46 \tag{12}$$

For RHEDA 2000 slab track system:

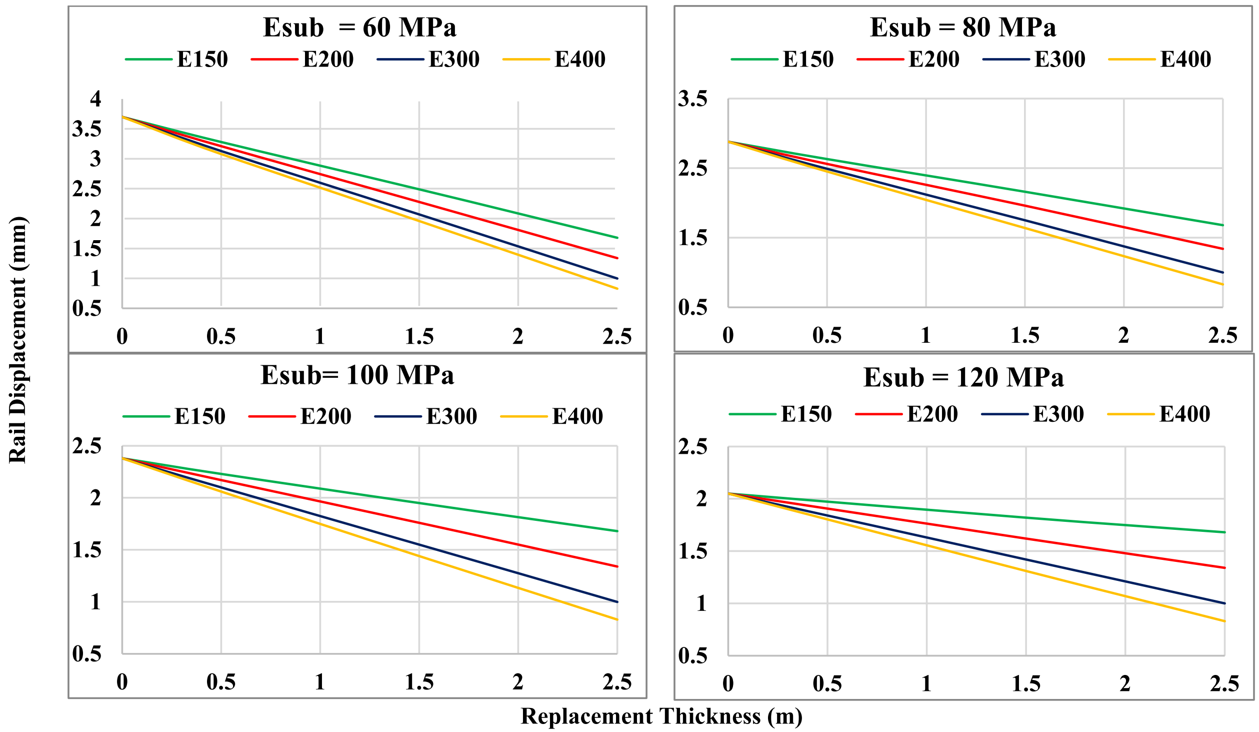

Fig. 16 Relationship between elasticity modulus of replacement soil and maximum rail deflection for different subgrade soils (different Esub) and thicknesses of replacement soil layer for RHEDA 2000 slab track (EN)

$$y_o = 166.96 \times E_{sub}^{-0.803} \tag{13}$$

$$m = 1.38\times10^{-7}\times E_{rep}^{3}-1.19\times10^{-4}\times E_{rep}^{2}+3.4\times10^{-2}\times E_{rep}+2.85\times10^{-3}\times E_{sub}^{2}-0.325\times E_{sub}+7.88 \tag{14}$$

The relationship between the rail displacement and modulus of elasticity for subgrade soil can be represented for BÖGL, Shinkansen, and RHEDA 2000 in Figs. 17, 18 and 19, respectively. These figures determine the minimum E_{sub} that does not require replacement soil; in this case, the minimum E_{sub} equals 62, 70, and 59.2 MPa for BÖGL, Shinkansen, and RHEDA 2000 slab track, respectively.

The general equations for BÖGL, Shinkansen, and RHEDA 2000 slab track (AREMA) have been obtained through regression analysis and can be represented graphically in Figs. 20, 21 and 22. This figure indicates the relationship between the vertical rail displacement in mm (y-axis), the elasticity modulus for subgrade soil in MPa (E_{sub} from 60 to 120 MPa) and elasticity modulus for replacement layer soil in MPa (E_{rep} from E150 to E400 MPa), and the thickness of replacement soils in meters (x-axis). The values between the lines shown in these three figures (Figs. 20, 21 and 22) can be easily integrated.

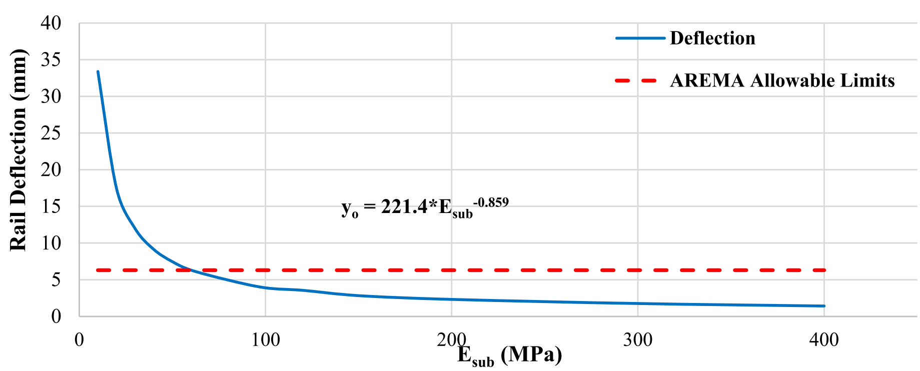

Fig. 17 Relationship between rail deflection (y_o) and subgrade elasticity modulus (E_{sub}) for BÖGL slab track (AREMA)

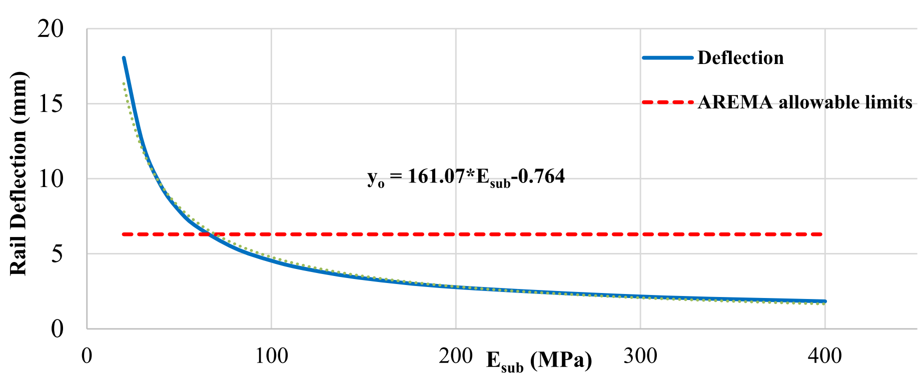

Fig. 18 Relationship between rail deflection (y_o) and subgrade elasticity modulus (E_{sub}) for Shinkansen slab track (AREMA)

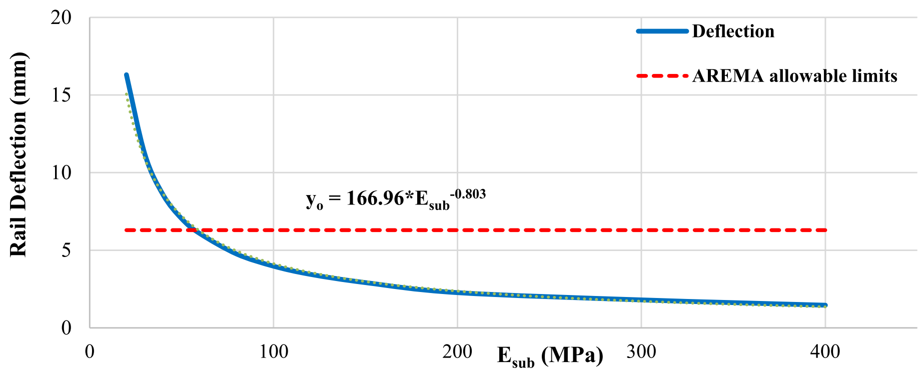

Fig. 19 Relationship between rail deflection (y_o) and subgrade elasticity modulus (E_{sub}) for RHEDA 2000 slab track (AREMA)

Conclusions

In this paper, a new design methodology for the slab track systems has been developed through the regression analysis for the parametric study of 300 FEMs for both AREMA and EN specifications. This methodology can be considered as the 1st design aid for the slab track systems. From the conducted studies, the following facts have been drawn:

1. The proposed new methodology is applicable to design slab track systems regarding rail deflection as this design methodology has been validated through experimental testing. A new design aid is presented using design equation and design graphs for the most used slab track systems.
2. The relationship between the rail deflection for slab track and the modulus of elasticity for subgrade soil (without any replacement layer) is described by a power function (nonlinear relationship) with a regression factor of R^2 equal to 0.98.

$$y_o = \beta \times E_{sub}^{-\alpha}$$

where "y_o" is the rail deflection (mm) considering the subgrade soil only and "E_{sub}" represents the modulus of elasticity of subgrade soil. Parameters "α" and "β" have been determined according to each slab track type by power regression analysis.

3. The relationship between the deflection (y in mm) at the top surface of the rail, the elasticity modulus of subgrade soil (E_{sub} in MPa), the elasticity modulus of replacement soil (E_{rep} in MPa), and the thickness of the replacement soil (d in meters) can be described as follows:

$$y = y_0 - m \times d$$
$$m = function\ of\ (a_i, E_{rep}, E_{sub})$$

where "y_o" is the rail deflection (mm) considering the subgrade soil only and the term "m" represents the subgrade and replacement soil characteristics (E_{sub} and E_{rep}). Parameter "a_i" has been evaluated according to each slab track type through polynomial regression analysis.

- For EN specifications, "m" can be obtained from the following model:

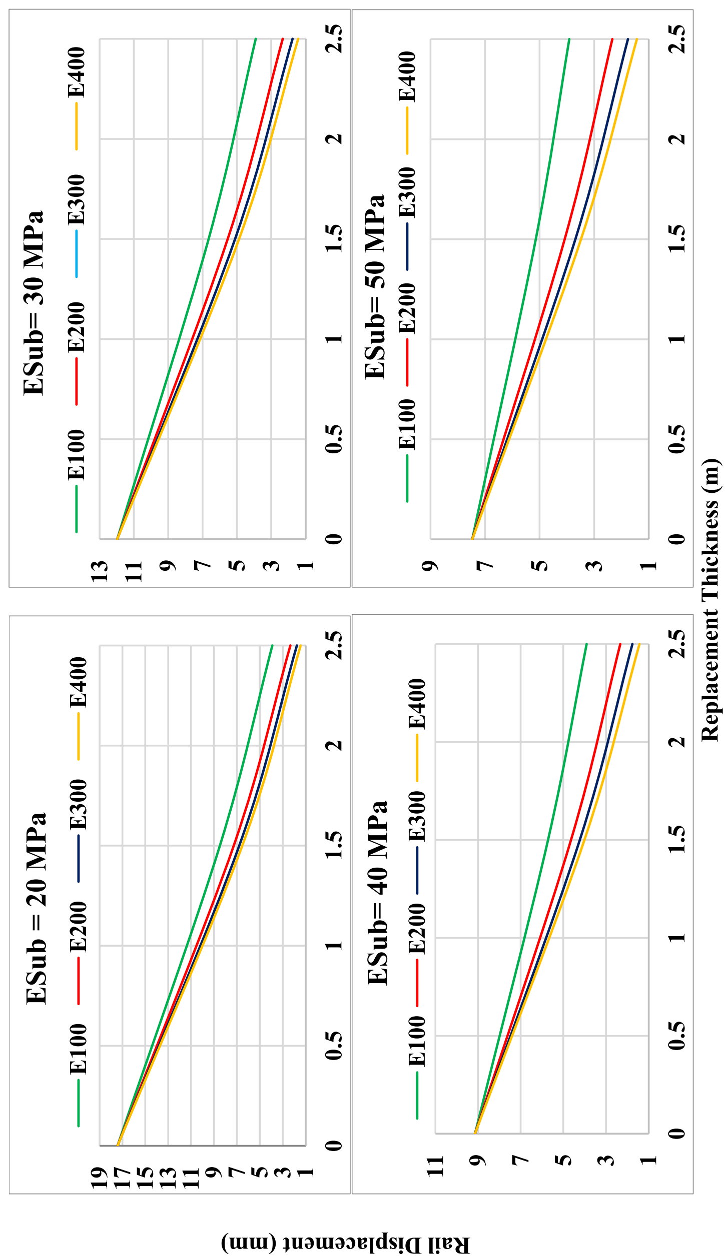

Fig. 20 Relation between elasticity modulus of replacement soil and maximum rail deflection for different subgrade soils and thicknesses of replacement soil layer for BÖGL slab track (AREMA)

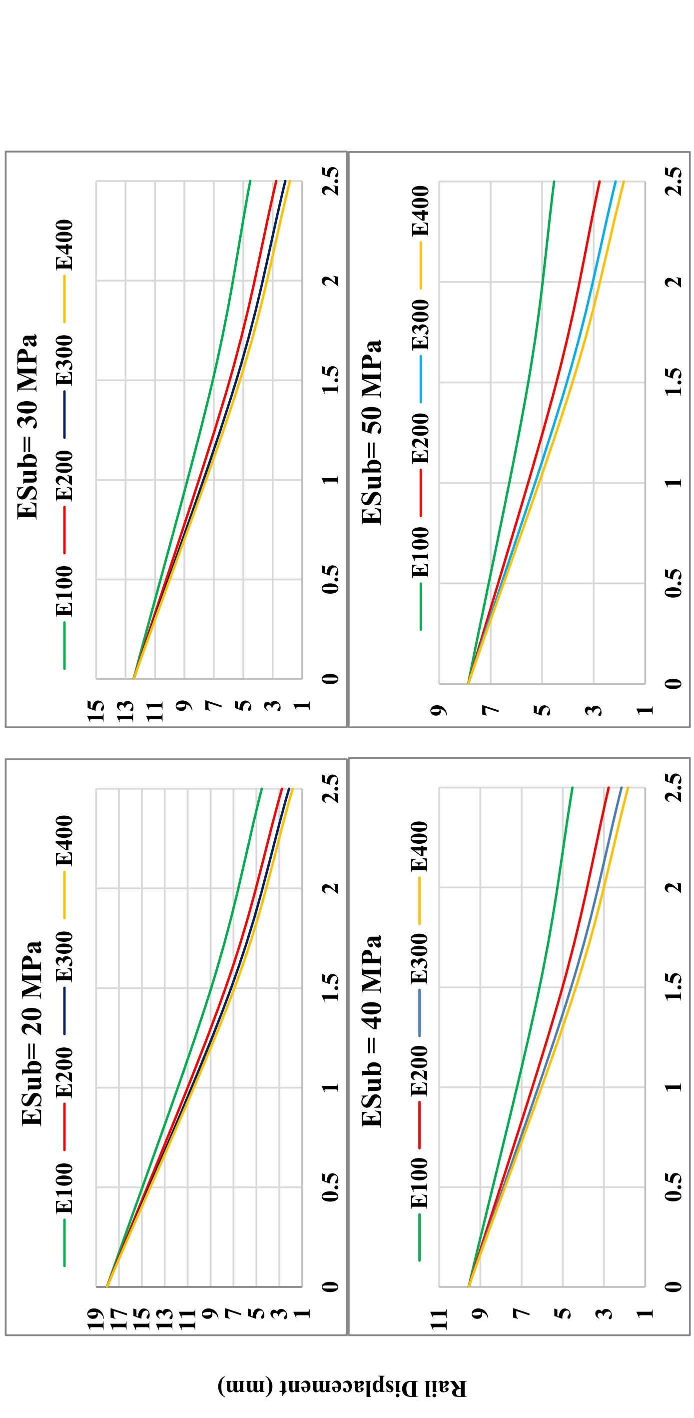

Fig. 21 Relation between elasticity modulus of replacement soil and maximum rail deflection for different subgrade soils and thicknesses of replacement soil layer for Shinkansen slab track (AREMA)

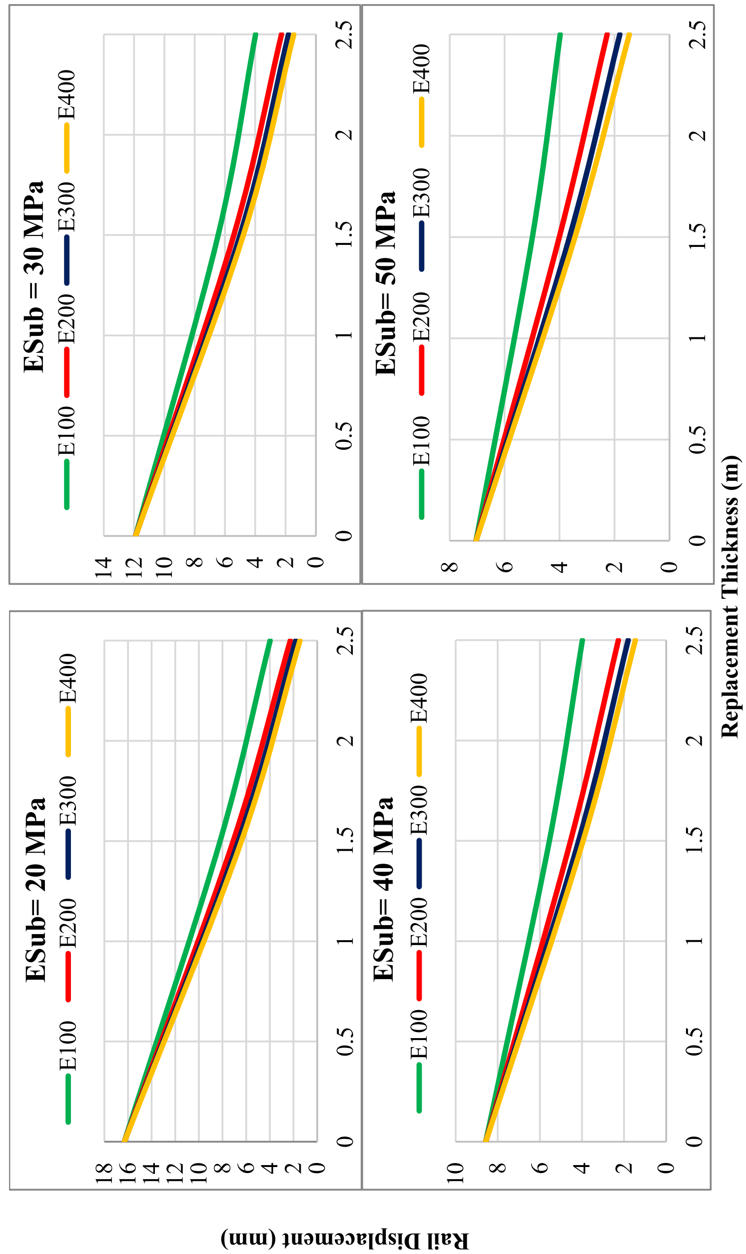

Fig. 22 Relation between elasticity modulus of replacement soil and maximum rail deflection for different subgrade soils and thicknesses of replacement soil layer for RHEDA 2000 slab track (AREMA)

$$m = -a_1 \times 10^{-6} \mathrm{E}_{rep}^2 - a_2 \times 10^{-3} \mathrm{E}_{rep} + a_3 \times 10^{-4} \mathrm{E}_{sub}^2 - a_4 \mathrm{E}_{sub} + a_5$$

◦ For AREMA specifications, "m" can be obtained from the following model:

$$m = a_1 \times 10^{-7} \mathrm{E}_{rep}^3 - a_2 \times 10^{-4} \mathrm{E}_{rep}^2 - a_3 \times 10^{-3} \mathrm{E}_{rep} + a_4 \times 10^{-4} \mathrm{E}_{sub}^2 - a_5 \mathrm{E}_{sub} + a_6$$

4. EN parametric study has proved that the minimum modulus of elasticity for subgrade soil to fulfill the allowable rail deflection criterion without a replacement soil should be taken as 130, 143, and 128 MPa for BÖGL, Shinkansen, and RHEDA 2000, respectively. In other words, the subgrade soil such as well-graded sand (E 127 MPa) or gravel (E 160 MPa) can be used as slab track subgrade without replacement soil for EN specifications.
5. AREMA parametric study has indicated that the minimum modulus of elasticity for subgrade soil to fulfill the allowable rail deflection limits without a replacement soil should be taken as 62, 70, and 59.2 MPa for BÖGL, Shinkansen, and RHEDA 2000, respectively. In other words, subgrade soil such as silty sand (E 60 to 70 MPa) can be used as slab track subgrade without replacement soil for AREMA specifications.
6. This study provides practical engineers with a simple design/analysis graph for each slab stack system according to the used design code and specifications.

Abbreviations

EN	European norms
AREMA	American Railway Engineering and Maintenance of the Way Association
CBL	Concrete-bearing layer
HBL	Hydraulic-bonded layer
FPL	Frost protective layer
CAM	Cemented asphalt mixture
GRAFT	Geo-pavement and Railways Accelerated Fatigue Testing
V	Train speed in km/h (EN) or mph (AREMA)
D_{33}	Diameter of a standard train wheel of 33 inch
D	Diameter of the actual train wheel in inches
γ	Soil unit weight in kN/m^3
E	Modulus of elasticity in MPa
C	Soil cohesion in kPa
υ	Poisson's ratio (unitless)
Φ	Angle of internal friction for soils
FEM	Finite element model
E_{rep}	Replacement soil modulus of elasticity in MPa
E_{sub}	Subgrade soil modulus of elasticity in MPa
Y_o	The rail deflection in subgrade soil only in mm
Y	The total rail deflection in the subgrade and replacement layer in mm
m	The slope for the linear regression equation which represents the subgrade and replacement soil characteristics
α and β	Regression parameters depending on the slab track type

Acknowledgements
Not applicable.

Authors' contributions
MHA, conceptualization, methodology, and reviewing and editing. IMA, supervising and editing. AAE, methodology, modelling, interpretation, and writing and editing. MAD, conceptualization, methodology, interpretation, and reviewing and editing.

Funding
No funding was obtained for this study.

Availability of data and materials
The datasets used and/or analyzed during the current study available from the corresponding author on reasonable request.

Declarations

Consent for participation
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Received: 3 November 2023 Accepted: 16 January 2024
Published online: 14 February 2024

References
1. Bilow DN, Randich GM (2000) Slab track for the next 100 years. In: AREMA proceedings of the 2000 annual conference American Railway Engineering and Maintenance-of-Way Association
2. Lichtberger B (2011) Track compendium. PMC Media House, Hamburg
3. Soliman AA (2023) Investigation of slab track design models. Transportation Department, Alexandria, p 212
4. Michas G (2012) Slab track systems for high-speed railways, KTH, School of Architecture and the Built Environment (ABE), Transport Science, Highway and Railway Engineering. Sweden. Retrieved from https://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-96507
5. Darr E, Fiebig W (2006) Feste fahrbahn: konstruktion und bauarten für eisenbahn und straßenbahn, 2nd edn. Eurail press, Hamburg
6. Darr E (2000) Ballastless track: design, types, track stability, maintenance and system comparison. Railw Tech Rev 3:36–45
7. Tayabji S (2000) Concrete slab track for freight and high speed service applications. A survey of practice. https://doi.org/10.1115/JRC2010-36046
8. Ando K, Sunaga M, Aoki H, Haga O (2001) Development of slab tracks for Hokuriku Shinkansen line. Q Rep RTRI 42(1):35–41
9. Wang T, Luo Q, Liu J, Liu G, Xie H (2020) Method for slab track substructure design at a speed of 400 km/h. Transp Geotech 24:100391
10. Atalan M, Prendergast LJ, Grizi A, Thom N (2022) A review of numerical models for slab-asphalt track railways. Infrastructures 7(4):59
11. Khajehdezfuly A, Poorveis D, Amiri AM (2023) Effect of track flexibility on fatigue life of railway concrete slab track. Constr Build Mater 382:131341. https://doi.org/10.1016/j.conbuildmat.2023.131341DOI
12. Čebašek TM, Esen A, Woodward PK, Laghrouche O, Connolly DP (2018) Full scale laboratory testing of ballast and concrete slab tracks under phased cyclic loading. Transp Geotech 17:33–40. https://doi.org/10.1016/j.trgeo.2018.08.003DOI
13. Ramos A, Correia AG, Calçada R, Costa PA, Esen A, Woodward P, Connolly D, Laghrouche O (2021) Influence of track foundation on the performance of ballast and concrete slab tracks under cyclic loading: physical modelling and numerical model calibration. Constr Build Mater 277:122245. https://doi.org/10.1016/j.conbuildmat.2021.122245DOI
14. Thölken D, AbdallaFilho JE, Pombo J, Sainz-Aja J, Carrascal I, Polanco J, Esen A, Laghrouche O, Woodward P (2021) Three-dimensional modelling of slab-track systems based on dynamic experimental tests. Transp Geotech 31:100663. https://doi.org/10.1016/j.trgeo.2021.100663. https://www.sciencedirect.com/science/article/pii/S2214391221001537
15. Cui X, Ling X (2021) Effects of differential subgrade settlement on damage distribution and mechanical properties of CRTS II slab track. Constr Build Mater 271:121821. https://doi.org/10.1016/j.conbuildmat.2020.121821DOI
16. Aly MHF, Elnaga IMA, Soliman AA, Diab MA (2023) Modelling of slab track systems for design purposes. J Eng Appl Sci 70(1):115. https://doi.org/10.1186/s44147-023-00277-7DOI
17. Aly MHF, Elnaga IMA, Soliman AA, Diab MA (2023) Linear and nonlinear modelling techniques for slab track systems. In: International Conference Advances in Structural and Geotechnical Engineering (ICASGE). International Conference of Advances in Geotechnical and Structural Engineering (ICAGSE 23), Hurghada
18. Esen AF, Laghrouche O, Woodward PK et al (2024) Numerical analysis of high-speed railway slab tracks using calibrated and validated 3D time-domain modelling. Rail Eng Sci 32:36–58. https://doi.org/10.1007/s40534-023-00315-3
19. Eurocode B (2004) Eurocode 2: design of concrete structures—. In: Part. p 230
20. Doyle NF, A.B.o.T. Economics (1980) Railway Track Design: A Review of Current Practice. Australian Government Publishing Service, Canbera. Retrieved from: https://www.bitre.gov.au/sites/default/files/op_035.pdf
21. Rakoczy AM, Nowak AS (2018) Live load spectra for railway bridges in USA. In: Konferencja Naukowa. Conference: 64 Scientific Conference, Railway engineering – challenges and opportunities, Poland
22. AREMA (1996) AREMA manual for railway engineering. American Railway Engineering Association, Washington DC
23. MIDAS G (2019) GTS NX user manual. MIDAS Information Technology Corporation Ltd., Seongnam. [in Korean]

Publisher's Note