Gkountakou and Papadopoulos
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The use of fuzzy linear regression for the selection of the most appropriate fuzzy implication in a fly ash-based concrete model

Fani Gkountakou[1] and Basil Papadopoulos[1*]

*Correspondence:
papadob@civil.duth.gr

[1] Department of Civil Engineering, Democritus University of Thrace, 67100 Xanthi, Greece

Abstract

In this research, fuzzy linear regression (FLR) method combined with three well-known fuzzy implications was implemented for evaluating the relation among the amount of fly ash in concrete mixture and the compressive strength of concrete. More specifically, 267 experimental data 40 of which were used for testing the validation of the process were subjected to FLR method for calculating the truth values, which indicated the degree of how the experimental outputs belong to the predicted ones. Also, the degree of fuzziness was calculated for performing the sensitivity analysis of the model. The truth values that emerged were used for applying three basic fuzzy implications such as Lukasiewicz, Reinchenbach, and Kleene-Dienes implication. By evaluating and comparing the results of every fuzzy implication, it was concluded that Lukasiewicz was the most appropriate implication method as it yielded the smallest deviation of truth values ($\sigma = 4.00$) in contrast to the theoretical ones ($\sigma = 4.83$ in Reinchenbach and $\sigma = 12.31$ in Kleene-Dienes fuzzy implication). The accuracy of the FLR method was also validated for calculating the coefficient of the mean absolute percentage error level (MAPE = 5.56%) of the blind prediction process, and the results revealed that the application of fuzzy linear regression method is suitable for evaluating the truth values of experimental data in order to be used in fuzzy implications. Thus, it is a satisfactory procedure for making inferences between concrete parameters.

Keywords: Fuzzy linear regression (FLR), Triangular fuzzy numbers, Fuzzy implications, Fly ash-based concrete, Approximate reasoning

Introduction

Concrete is one of the most commonly used materials worldwide for construction purposes. Its usage is essential in buildings, roads, and many civil constructions as its raw materials are cheap and locally available. Also, it is known for its distinctive properties such as high durability, consistency, and modularity [1]. However, its production caused significant amounts of CO_2 emissions resulting in damaging the natural environment [2]. Therefore, the use of waste materials gained the attention of many researchers who are interested in protecting the environment from cementitious materials, which enhance the mechanical characteristics of modified concrete.

Many studies referred to replace ordinary Portland cement with waste materials [3]. For example, rice husk ash [4] was used as a sustainable waste material for mitigating the CO_2 emission and enhance the service life of the sustainable construction. Also, Ullah et al. [5] used lightweight foamed concrete as an adaptable generation of eco-friendly materials. The replacement of a certain amount of concrete with entrapped air bubbles led to a more sustainable material with improved mechanical characteristics.

One of the most important factors that characterizes the stability of concrete is its compressive strength, as it is essential for assessing the structures' performance. Although the evaluation of the compressive strength of concrete is a complex method affected by many factors, it is vital to determinate it for ensuring the structures' safety by carrying out experimental procedures under laboratory conditions. Lately, many studies claimed that optimization models using machine learning were capable of predicting the compressive strength of concrete avoiding time consuming tests [6–9]. Particularly, random forest regression and gene expression programming were applied for evaluating the compressive strength of fly-ash-dependent geopolymer concrete and high strength concrete [10, 11]. The results revealed that the last method provided high performance and an empirical expression of the model. Also, Nafees et al. [12] used multilayer perceptron neural networks, adaptive neural fuzzy detection systems, and genetic programming for forecasting the compressive strength of silica fume-based green concrete. It was found that the first method yielded the most accurate predicted results than the other methods, and it was a suitable method for concrete construction models. Machine learning was also applied with the algorithm named "multi-expression programming" for predicting the compressive strength of carbon fiber-reinforced polymer confined concrete [13]. The RMSE parameter confirmed the accuracy of the model as the values of the database, and the training sets were 7.68 and 7.76, respectively, which indicated that RMSE parameters for both cases were close. In addition, three methods such as gene expression programming, multiple linear regression, and multiple nonlinear regression were used for estimating the compressive strength of bagasse ash-based concrete [14] and it was demonstrated that gene expression programming had the lowest values of RMSE coefficient in the calibration and validation process.

The novelty of this research involves the evaluation of the relation among the amount of fly ash in concrete mixture and the compressive strength of concrete with the use of three well-known fuzzy implications via fuzzy linear regression (FLR) method. Fuzzy implications are widely implemented for solving fuzzy conditionals involving fuzzy statements, with the form of "If p, then q" rule. Particularly, the function of fuzzy implications which is formed as I: [0,1] x [0,1] → [0,1], indicates that the degree of truth of the conditional is expressed from the initial statements [15]. Thus, its utilization is necessary for approximate reasoning as it provides an algorithmic solving procedure.

Fuzzy implications demonstrate extension of classical implications from the boundaries {0,1} to the boundaries [0,1], in which the truth values belong. More specifically, a classical implication is defined as m: {0,1} x {0,1} → {0,1} that implies that the truth values are equal only to 1 or 0, which means that the proposition is either true or false, in contrast to fuzzy implications in which the truth values are between the full interval [0,1] [16]. Even though fuzzy implications are expressed with various definitions in the literature, in this research, they are defined with the following theorem [17–19]:

Theorem 1 *A function I:* [0,1] *x* [0,1] → [0,1] *is called fuzzy implication which determines the degree of truth of any fuzzy proposition and must satisfy the following axioms:*

$$x_1 \leq x_2 then I(x_1, y) \geq I(x_2, y), i.e, I(\cdot, y) \text{ is decreasing} \tag{1}$$

$$y_1 \leq y_2 then I(x, y_1) \leq I(x, y_2), i.e, I(x, \cdot) \text{ is increasing} \tag{2}$$

$$I(0, 0) = 1 \tag{3}$$

$$I(1, 1) = 1 \tag{4}$$

$$I(0, 1) = 1 \tag{5}$$

$$I(1, 0) = 0 \tag{6}$$

in which x_1, x_2, x, y_1, y_2, y are the truth values and belong to the domain [0,1]. The axioms (3) to (6) are a generalization of classical implication properties. A fuzzy implication is also said to obey the following conditions [20, 21]:

$$I(1, y) = y, y \in [0, 1], \text{which is the left neutrality property} \tag{7}$$

$$I(x, I(y, z)) = I(y, I(x, z)), x, y, z \in [0, 1] \text{ which is the exchange principle property} \tag{8}$$

$$I(x, x) = 1, x \in [0, 1] \text{ which is the identify principle property} \tag{9}$$

$$I(x, y) = 1 \Leftrightarrow x \leq y, x, y \in [0, 1] \text{ which is the ordering property} \tag{10}$$

The distinctive quantities of fuzzy implications have attracted the attention of many researchers with devotion to fuzzy implication functions. For example, Mas et al. [15] analyzed the utilization of fuzzy implications to many fields like approximate reasoning, fuzzy relational equations, fuzzy mathematical morphology, and fuzzy control. Also, fuzzy implications were used to make backward and forward inferences with the application of fuzzy conditions. Particularly, Mylonas et al. [22] proposed a method for evaluating the most appropriate implication which was applied to a set of data (x_i,y_i). In the first step, the membership value of each variable was determined and the max or centroid defuzzification methods were applied for computing the output $\widehat{y}_i$. Then, the deviations of every implication method from the initial observations were calculated, with the following form:

$$d = \sqrt{\frac{1}{n} \sum \left(\widehat{y}_i - y_i\right)^2} \tag{11}$$

in which $\widehat{y_i}$ are the outputs that emerged from the implication method and y_i are the observations. By applying this procedure for every fuzzy implication, it was concluded that the Lucasievich and Godel implications had the smallest deviations and thus were the most satisfied implication methods. Also, a new empiristic implication method [23, 24] was proposed based on well-known fuzzy implications. Firstly, fuzzy C-means clustering (FCM) algorithm was used for categorizing the data into ascending order and therefore for setting language variables. Then, the Sturges formula was used for dividing the observations into classes and the empiristic implication was applied through fuzzy negations for determining the most appropriate implication method.

The purpose of our study is to estimate the relation among the amount of fly ash in concrete mixture and the compressive strength of concrete with the use of three well-known fuzzy implications via fuzzy linear regression (FLR) method. More specifically, Botzoris et al. [25] claimed that by applying the fuzzy implication $x \Rightarrow y$ in a set of data based on real observations (x,y), and the truth value was equivalent to one. Thus, by using FLR with triangular fuzzy numbers, the degree of truth of every fuzzy output was evaluated, which turned to be the truth value of the total fuzzy implication. This reasoning was based on the aforementioned fuzzy implication condition as presented in Eq. (7). Taking this into account, 267 dataset that were studied in previous paper [26], 40 of which were used for testing the validation of the method, were subjected to FLR method in two different cases for evaluating the truth value of every observation. In the first one, fly ash was used as an input variable and the compressive strength of concrete was used as an output and in the second one the compressive strength of concrete was the independent variable and fly ash was the output. The results that emerged from the aforementioned cases were applied to three well-known fuzzy implications such as Lukasiewicz [27], Reinchenbach [28], and Kleene-Dienes [29]. Finally, the deviations of every implication method were calculated, and thus, the most satisfied fuzzy implication was selected.

The parts of this study are the followings: the second section quotes the properties of concrete with fly ash admixture and the third section describes the data emerged from the literature and evaluates the Pearson coefficient. The fourth part depicts the use of fuzzy linear regression method for making backward and forward inferences. Then, the fifth part analyzes the application of this process for evaluating the truth values that emerged from the relation between the fly ash content and the compressive strength of concrete and the sensitivity analysis process. In conclusion, the last part summarizes the results and highlights the use of FLR method in the implication process.

Fly ash-based concrete

As the population growth and the development of the civilization increase, the demand of sustainable concrete seems to be necessary for reducing global CO_2 emission [30–32]. For that reason, the ordinary Portland cement can be replaced with fly ash, which is a coal combustion by-product with improved mechanical properties [33, 34]. This replacement not only contributes to decrease the environmental impact but also to enhance the workability and durability of concrete [35].

The determination of the mechanical properties of fly ash concrete is essential for its efficient use and for avoiding concrete failures in constructions. Thus, fly ash concrete has attracted the interest of many researchers who conducted numerous studies about its effect on the mechanical properties of concrete, that are discussed in the following sub-sections.

Compressive strength (MPa)

The compressive strength is the most important property as it affects the constructive behavior of concrete. More specifically, it was reported that the replacement of ordinary cement with 10% fly ash increased the 90-day compressive strength of concrete, as well as the proportion of 20% and 30% fly ash yielded higher compressive strength at 180 and 365 days, respectively [36]. Similarly, it was reported that high proportions of fly ash increased the long-term compressive strength of concrete [37]. In addition, Harison et al. [38] observed that by replacing 20% Portland Pozzolana Cement with fly ash, the compressive strength was increased from 1.9 to 3.28%, at the age of 28 and 56 days.

Flexural strength (MPa)

Many researchers claimed that the replacement of ordinary Portland cement with fly ash turned out to be an effective way for increasing the flexural strength of concrete. For example, Atiş [39] observed that the mixture with 50% fly ash revealed 5.21 and 6.11 MPa at 7 and 28 days of hydration in contrast to the reference specimen which had 4.43 and 5.37 MPa, respectively. Also, Kumar et al. [40] studied the replacement of ordinary Portland cement with 20%, 30%, 40%, 50%, and 60% fly ash. It was found that the proportions of 20%, 30%, 40%, and 50% fly ash yielded higher flexural strength than the reference specimen at 180, 256, and 365 days. It was also investigated [37] that the proportions of samples with 20%, 40%, and 60% fly ash gained 7.2, 7.1, and 7.5 MPa flexural strength at 365 days of hydration compared to the concrete without fly ash, the value of which was 6.5 MPa.

Splitting tensile strength (MPa)

Splitting tensile strength is also a key parameter that must be determined for the characterization of concrete utilization. For optimizing the splitting tensile strength, 50% fly ash replaced the ordinary Portland cement [39] and the results revealed that the splitting tensile strength increased in 28 days of hydration. Furthermore, three different proportions of fly ash were studied for determining the mechanical properties of fly ash concrete [41]. It was concluded that 50% fly ash yielded better splitting tensile strength results compared to the other two mixes which had 60% and 65% fly ash content, respectively. More specifically, at 28 days of hydration the splitting tensile strength was 3.5 MPa for the mixture containing 50% fly ash and for the other two samples the splitting tensile strength values were 3.1 and 2.8 MPa, respectively.

Data description

Waste materials are essential in construction industry for mitigating the environmental hazards and improving the mechanical properties of concrete in a sustainable engineering process [42–44]. Fly ash is one of the most commonly used waste materials produced by powered coal [45]. Its pozzolanic activity brings numerous benefits in concrete mixture such as improving the workability and the compressive strength of concrete, reducing the heat in the hydration process, decreasing permeability, and increasing the durability in harsh environment [46]. The accurately prediction of the compressive strength of fly ash concrete is a complex method as it affects many parameters [47]. However, this process is vital for characterizing the concretes' performance.

In order to develop accurate prediction model for evaluating the compressive strength of fly ash concrete and then calculating the relation about parameters with three well-known fuzzy implications, a series of 267 data sets was used for training and test procedures retrieved form a previous study [26]. The data are summarized in Supplementary Table 1. In Fig. 1, a and b are the violin graphs which represent the shape of the data set. These graphs illustrate the distribution of the dependent and independent variables, the density curves of every group and the way that the data are distributed around the median.

As it is shown in Fig. 1, the median values of fly ash and compressive strength of concrete were 162 kg/m^3 and 35.3 kg/m^3, respectively.

For determining the linear correlation between fly ash input and the compressive strength of concrete, a statistical coefficient named Pearson correlation was calculated as follows:

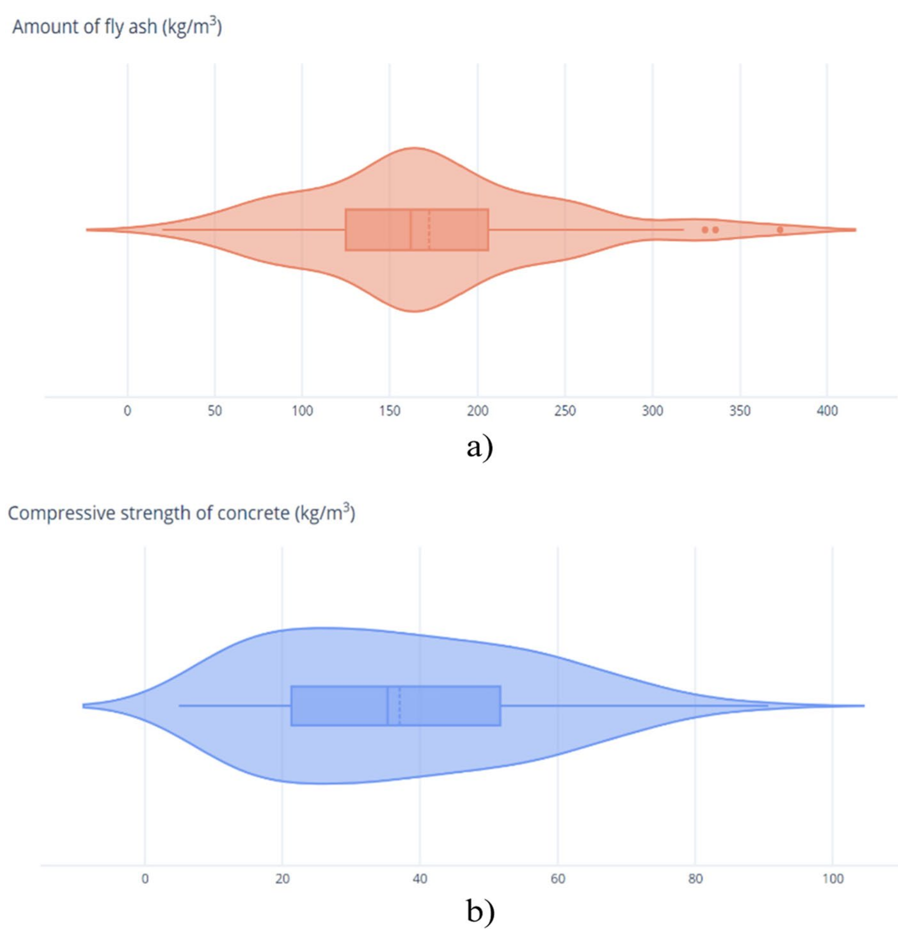

Fig. 1 Violin graphs of the **a** amount of fly ash and **b** compressive strength of concrete

$$r_{XY}=\frac{n\cdot\sum_{i=1}^{n}x_i\cdot y_i-\left(\sum_{i=1}^{n}x_i\right)\cdot\left(\sum_{i=1}^{n}y_i\right)}{\sqrt{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}\cdot\sqrt{n\sum_{i=1}^{n}y_i^2-\left(\sum_{i=1}^{n}y_i\right)^2}}=-0.013 \quad (12)$$

The value of the Pearson coefficient that emerged indicated that the correlation between the parameters was negative low. Therefore, fuzzy linear regression is the most suitable method for studying these data that emerged from laboratory procedures, as this method uses fuzzy parameters for defining the relation between the variables instead of statistical concept [48]. On the contrary, linear regression is not recommended in such cases, in which there is a vagueness between the variables.

Methods

Fuzzy linear regression method

Classical linear regression analysis is an effective method for making statistical assumptions between one depended variable and one or more independent inputs [49, 50]. However, the randomness that this method is based on makes the prediction process uncertain, mitigating the utilization of linear regression. Fuzzy linear regression (FLR) is an extension of classical to fuzzy logic which constructs the relationship among variables in a fuzzy environment. This method was first proposed by Tanaka [51] who introduced the meaning of the fuzziness of the model and claimed that the deviations among the observed and the estimated values were due to the vagueness of the model and were expressed with fuzzy numbers. The equation of the FLR method had the following form [52–54]:

$$Y=\mathrm{A}_0+\mathrm{A}_1\mathrm{X}_1+\cdots+\mathrm{A}_nX_n \quad (13)$$

in which X_n are the inputs, Y is the output, and A_n are the fuzzy numbers. In our study, we used symmetric triangular fuzzy numbers the membership function of which was [55]:

$$\mu_{\mathrm{A}}(x)=L\left(\frac{x-r}{c}\right),c>0 \quad (14)$$

where c and r are the spread and the center of the fuzzy number, respectively, and $L(x)$ is a shape function that satisfied the following constraints [56]:

$$L(x)=L(-x) \quad (15)$$

$$L(0)=1 \quad (16)$$

$$L(1)=0 \quad (17)$$

$$L(x)\text{ is decreasing on }[0,\infty) \quad (18)$$

$$L(x)\text{ is invertible on }[0,1] \quad (19)$$

As the fuzzy numbers were triangular, the reference function $L(x)$ had the following form [57]:

$$L(x) = max(0, 1 - |x|) \tag{20}$$

By applying FLR method with fuzzy numbers, the outputs Y_j that emerged were also fuzzy triangular numbers with the following membership function [58]:

$$\mu_{Y_j}(y_j) = L\left[\frac{y_j - (r_0 + \sum_{i=1}^{n} r_i x_{ij})}{c_0 + \sum_{i=1}^{n} c_i |x_{ij}|}\right] \tag{21}$$

Then, a degree of certainty $0 \le h < 1$ was set in which the available data wish to be included in the estimated variable Y_j [59, 60].

$$\mu_{Yj}(y_j) \ge h \tag{22}$$

For evaluating the values of the fuzzy coefficients $\mathrm{A}_i = (r_i, c_i)$ and optimizing the fuzzy predictive model, the following linear programming problem was emerged [61]:

$$J = min\{mc_0 + \sum_{j=1}^{m} \sum_{i=1}^{n} c_i |x_{ij}|\} \tag{23}$$

$$y_j \ge r_0 + \sum_{i=1}^{n} r_i x_{ij} - (1-h)\left(c_0 + \sum_{i=1}^{n} c_1 |x_{ij}|\right) \tag{24}$$

$$y_j \le r_0 + \sum_{i=1}^{n} r_i x_{ij} + (1-h)\left(c_0 + \sum_{i=1}^{n} c_i |x_{ij}|\right) \tag{25}$$

$$c_i \ge 0, i = 1, 2, \ldots, n \tag{26}$$

in which the Eqs. (24)–(26) were applied for minimizing the objective function in Eq. (23).

Results and discussion

Fuzzy linear regression (FLR) is an alternative approach of specifying the deterministic relationship between independent variables and one dependent output. Its use is widespread for setting a fuzzy approximation theorem system which minimizes the deviations of observed and predicted coefficients. Also, this method deals with the vagueness that is involved between the variables, as it uses fuzzy numbers that express the ambiguity of the model. In our study, for making backward and forward inferences for the degree of truth between the amount of fly ash in concrete mixture and the compressive strength of concrete, FLR analysis was used twice. In the first case, the amount of fly ash was the input variable and the compressive strength was the output and in the second case the compressive strength of concrete was the independent variable and the amount of fly ash was the output.

It is well known that many researchers were interested in protecting the environment from the emission of CO_2 gases. The manufacturing of Portland cement emits amounts of CO_2 in the atmosphere resulting in greenhouse gases [62]. Thus, the development of ecofriendly cement-based materials is vital in minimizing this cause.

Fly ash is a sustainable substitute for ordinary Portland cement with many benefits in construction industry. The study of its mechanical properties is very important for ensuring the durability of the concrete and thus, the construction safety. Therefore, the determination of the effect of the amount of fly ash in the compressive strength of concrete is necessary for safety reasons.

Several studies analyze the utilization of predicting the compressive strength of fly ash-based concrete [48, 63]. In a previous study [26], four different methods based on machine learning such as artificial neural network, random forest, decision tree, and gradient-boosting network were applied for estimating the compressive strength of fly ash concrete. The data were emerged from the literature. In our research, the same 227 sets of data were applied for evaluating the degree of truth between the amount of fly ash and the compressive strength of fly ash-based concrete at different days of hydration. Then, these results were subjected into three different fuzzy implications for determining the most appropriate relation between the parameters.

In the first case, FLR analysis was used basis on Eq. (13):

$$Y = \mathrm{A}_0 + \mathrm{A}_1 X \tag{27}$$

where Y was the compressive strength of fly ash concrete (MPa) and X was the amount of fly ash (kg/m^3). For minimizing the spread of the fuzzy numbers, the linear programming problem was solved with the following objective function, by using the simplex method:

$$J = min\left\{ mc_0 + \sum_{j=1}^{227} c_1 |x_{1j}| \right\} \tag{28}$$

In the second case, X was the compressive strength of concrete and Y was the amount of fly ash, and the same procedure was applied. The results of the fuzzy coefficients for every case were summarized in Table 1.

Furthermore, from analyzing the results of fuzzy outputs, the truth values of every set of data that determine to what degree the observed data belong to the predicted ones were calculated. The truth values $\mu_A(y_i)$ of membership function were defined into the interval [0,1]. In addition, a blind prediction of the compressive strength was applied for testing the validation of the FLR method. For that reason, a set of 40 data were implemented for evaluating the compressive strength of fly ash concrete. The aforementioned results of fuzzy linear regression method of blind prediction process were reported in Supplementary Table 2.

Table 1 The results of the fuzzy triangular numbers

	First case		Second case	
Variable	**Estimate R_i**	**Estimate C_i**	**Estimate R_i**	**Estimate C_i**
A_0	54.535	41.992	196.500	176.505
A_1	− 0.085	0.000	− 0.001	0.000

Table 2 The results of the fuzzy triangular numbers of sensitivity analysis model

	First case		Second case	
Variable	**Estimate R_i**	**Estimate C_i**	**Estimate R_i**	**Estimate C_i**
A_0	54.546	42.030	196.501	176.701
A_1	− 0.085	0.000	− 0.001	0.000

Table 3 The results of the degree of fuzziness

First case		Second case		
	FLR analysis model	**Sensitivity analysis model**	**FLR analysis model**	**Sensitivity analysis model**
$F(M^h)$	41.992	42.030	176.505	176.701

In order to evaluate the results that emerged from the blind prediction process, the values of the coefficient of the mean absolute percentage error (MAPE) were determined with the following equation:

$$MAPE = 100\% \frac{1}{n} \sum_{t=1}^{n} \left| \frac{e_t}{A_t} \right| = 100\% \frac{1}{n} \sum_{t=1}^{40} \left| \frac{A_t - F_t}{A_t} \right| = 5.56\% \tag{29}$$

in which A_t and F_t were the observed and estimated values of the compressive strength of concrete, respectively, and n was the number of sets.

For performing the sensitivity analysis and analyzing the relative significance of considered parameters, the coefficient of the degree of fuzziness was determined with the following equation:

$$F\left(M^h\right) = \sqrt{\left(c_0^h\right)^2 + \left(c_1^h\right)^2 + \cdots + \left(c_n^h\right)^2} \tag{30}$$

where c_n was the spread of the fuzzy numbers. More specifically, sensitivity analysis is used for assessing the alterations of parameter values in the model performance [64]. If the order of rankings remains constant, the findings obtained from the analysis can be considered robust and consistent. Conversely, if the criteria are found to be sensitive to the level of uncertainty, it indicates that the results are influenced by the degree of fuzziness in the values [65]. For that purpose, the values of the input and output parameters of the two aforementioned cases of the model were increased by 0.01 and were subjected to FLR method. The results of the fuzzy coefficients for every case were summarized in Table 2.

Then, for evaluating the degree of fuzziness for every model, Eq. (30) was used, and the results were represented in the following Table 3. In the first case, FLR analysis model was the aforementioned model in which Y was the compressive strength of fly ash concrete and X was the amount of fly ash and sensitivity analysis model was the model that it used the same parameters increased by 0.01. In the second case, FLR analysis model was the aforementioned model that the compressive strength of fly ash concrete was used as input and the amount of fly ash was the output and sensitivity analysis model was the increased model, respectively.

Table 4 Basic fuzzy implications

Fuzzy implications	Formula
Lukasiewicz	$I_{LK}(x, y) = \min(1, 1 - x + y)$
Reinchenbach	$I_{RC}(x, y) = 1 - x + xy$
Kleene-Dienes	$I_{KD}(x, y) = \max(1 - x, y)$

Table 5 The results of the deviations of every fuzzy implication

Fuzzy implications	Formula
Lukasiewicz	4.00
Reinchenbach	4.83
Kleene-Dienes	12.31

The results that emerged from Table 3 proved that the degree of fuzziness for both models of sensitivity analysis changed in low percentage, which indicated that the relationship between the input and output parameter was robust and stable.

By evaluating the aforementioned results, it was demonstrated that the use of FLR method emerged accurate predicted values, and thus, it is a validate method.

Taking into account the results of the values of membership functions that emerged from the application of FLR method, three different implications which were summarized in Table 4 were used:

in which x and y represent the truth values $\mu_A(y_i)$ that emerged from the first case and second case of FLR method, respectively.

The deviations that resulted from every implication method were calculated with the following equation:

$$\sigma = \sqrt{(1 - \lambda_1)^2 + (1 - \lambda_2)^2 + \cdots + (1 - \lambda_{227})^2} \tag{31}$$

in which λ is the truth value that emerged from the implications. This equation was based on the fact that in an ideal fuzzy system, the truth values of a fuzzy implication $x \Rightarrow y$ were equivalent to one, as data (x, y) were real observations [25]. Thus, the most appropriate implication method needed to have the smallest deviations from the value 1. The deviation results of every implication method were summarized in Table 5.

As indicated, the use of FLR method is not limited to prediction problems but it can be used effectively for determining the relation between parameters. Its equation that combines the inputs of every problem with fuzzy numbers provides the degree of truth of every set of data (x,y), which are applied in fuzzy implications. According to the Table 5, in this case, the best implication process that had effectively evaluated the relation among the amount of fly ash in concrete mixture and the compressive strength of concrete was the Lukasiewicz implication method. This was due to the fact that the Lukasiewicz implication yielded the smallest deviation ($\sigma = 4.00$) compared to the other implication methods.

One more advantage of the FLR method is that it determines the truth values of data without having to divide them into language parameters. More specifically, the separation of experimental data into linguistic parameters for determining the degree of truth of membership function for every fuzzy set is an uncertain process if there is no information about the data [18]. By using the fuzzy linear regression method as implication process, the identification of language parameters is unnecessary as the membership functions are calculated from the fuzzy outputs. In contrast to other predictive black box methods based on machine learning [66], such as artificial neural network [67, 68], adaptive neuro fuzzy interface system [68], gene expression programming [10, 68, 69], decision tree [69], and random forest [10], the FLR method uses a specific equation for evaluating the output. It also provides the equation of the membership function, and hence, the truth values of very set of data that are used for evaluating the most appropriate fuzzy implication. Therefore, FLR analysis is an accurate process for evaluating the truth values of data in order to develop approximate reasoning.

Conclusions

The theory of fuzzy implications is widely used for making inferences through approximate reasoning in a fuzzy rule environment. The aim of this study was to select the most appropriate fuzzy implication in a fly ash-based concrete model by combining fuzzy linear regression method with fuzzy implications. More specifically, fuzzy linear regression with triangular fuzzy numbers was used for determining the truth values of the amount of fly ash in concrete mixture and the compressive strength of fly ash-based concrete. Fuzzy linear regression is a widely used prediction method the utilization of which has been demonstrated for many researchers. The implication axiom based on neutrality of truth defined that the truth value of every fuzzy output turns to be the truth value of the total fuzzy implication. According to this fact, these values were subjected into three fuzzy implications such as Lukasiewicz, Reinchenbach, and Kleene-Dienes for defining the most appropriate implication method. By evaluating and comparing the results of every fuzzy implication, it was concluded that Lukasiewicz was the most appropriate implication method as it yielded the smallest deviation of truth values in contrast to the theoretical ones.

In addition, the application of FLR method for evaluating the relationship between concrete parameters deals with the uncertainties that are involved in separating data into language variables. More specifically, the FLR method gives an exact relation between the parameters and the results come up accurately without needing to divide data into linguistics parameters. This property is necessary in case where the information about the input variables is incomplete. In our study, the truth values that we used for evaluating the best implication process were emerged from FLR method accurately, and thus, the identification of linguistic variables was unnecessary as the membership functions were calculated from the fuzzy outputs. Also, the degree of fuzziness that emerged from applying the FLR method was used effectively from performing the sensitivity analysis of the model. The results revealed that the relationship between the input and output parameter was robust and stable.

In conclusion, the implication process of Lukasiewicz yielded the best results as the deviation was $\sigma = 4.00$ in contrast to the other methods the deviations of which were higher. Thus, FLR method proves to be an accurate prediction process for membership function evaluation and a promising procedure in approximate reasoning.

Abbreviations

FLR	Fuzzy linear regression
I	Fuzzy implication
d	Deviations
X_i	Input
Y	Output
A_i	Triangular fuzzy number
J	Objective function
$\mu_A(y_i)$	Truth values of membership function
r	Center of triangular number
c	Spread of triangular number
L(x)	Shape function
h	Degree of certainty
σ	Deviations from every implication method
λ	Truth values from implications
MAPE	Mean absolute percentage error
$F(M^h)$	Degree of fuzziness
r_{XY}	Pearson correlation

Supplementary Information

The online version contains supplementary material available at https://doi.org/10.1186/s44147-023-00266-w.

Additional file 1: Table S1. The prediction of the two cases by the FLR method through 227 observations. **Table S2.** The blind prediction of the two cases as evaluated by the FLR method through 40 observations.

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Authors' contributions
Conceptualization, methodology, software, validation, formal analysis, investigation, resources, data curation, preparation, and writing the draft of manuscript were done by FG under the supervision of BP. All authors read and approved the final manuscript.

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Availability of data and materials
All data generated or analyzed during this study are included in this published article.

Declarations

Competing interests
The authors declare that they have no competing interests.

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