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Availability and cost–benefit analysis of a fault tolerant series–parallel system with human-robotic operators

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Abstract

Without excellent system uptime and profit margins, many manufacturing systems will not be able to continue operating. Strong business performance of manufacturing companies is facilitated by system availability and profit generation. Most manufacturing systems are set up in series–parallel, parallel-series, or hybrid configurations. In this present study, we analyze a series–parallel system composed of two subsystems with the following specifications: subsystem A consists of two similar units/components that are operated by Human, whereas subsystem B is made up of two similar units/components that are operated by Robot. We have also introduced fault tolerance factor in this work, so that the failure of each unit in each subsystem, common cause failure as well as failure due to human error and robot, will be accompanied by this fault tolerance factor. Our key goals are to examine how fault tolerance will increase the model's availability and profitability and to identify optimum maintenance plan. In order to meet these key goals, certain expressions for reliability metrics have been developed and validated through numerical examples. Tables and graphs are used to illustrate the results and form conclusions from them.

Keywords: Availability, Fault tolerance factor, Common cause failure, Expected profit

Introduction

Without high system uptime (availability) and profit generation, many industrial companies will not be able to continue operating. A cost–benefit analysis, also referred to as benefit–cost analysis, is a systematic process that allows industries to evaluate decisions, systems, as well as determining the value of intangible assets. Cost–benefit analysis is a general technique that is frequently used in engineering. In many industries today, it is essential to make the most of idea and option. To achieve this end, many industries, from large to start-ups and small, use cost–benefit analysis to assist them make important and relevant decisions. Using cost–benefit analysis can assist many industries in determining the highest and expected value of a design/system. Even so, some additional value-laden assumptions and decision must be made in such cases. In general, the manager of an industry will seek to maximize profits for the industry, since profits are determined both by the revenue earned by the industry and the amount spent to operate the industry.

Profit can be increased by increasing revenue as well as decreasing operating expenses. Managers will frequently choose this method because it appears to be a simpler way of increasing profits.

In general, process industries are made up of sophisticated engineering systems or subsystems that are configured either in series, parallel, or hybrid configurations. Vast majority of these systems are operated by humans or robots. The use of robotics in industry is very widespread. Manufacturing products used to be a laborious process that needed a sizable workforce. It was challenging to control the pace of production since, in the past, every task was completed by hand (human). Today, many jobs that once required physical labor have been automated. This is one of the most well-known applications of robotics. However, failure is unavoidable, though there are other factors that can also lead to the failure of these systems. They include poor system design, overload, maintenance delays, lack of operational skills and common cause failure. These factors may contribute to the low or non-availability of the industrial systems, resulting in inefficient resource utilization. In order to be efficient, each system should operate without failure for an extended period of time. As a result, developing a suitable maintenance technique that will increase system availability and generates more profit will help in this direction.

Fault tolerance is the process of ensuring that a system/machine continues to function properly in the face of system failures. Even after thorough investigation/testing, there is still a chance that the system will fail. In practice, no system can be completely error-free. As a result, systems are designed in such a way that, in the event of error availability and failure, the system performs its function correctly and provides the desired result.

There are numerous situations in real life, in which multiple repairs between adjacent transition states are possible in order to quickly repair the failed system. When this occurs, the system is repaired with Copula. The copula technique is a powerful technique for describing interdependence among variables that has sparked a lot of interest in a variety of domains. The joint lifetime distribution can be generated by modeling component dependence with a copula function, making it more practical and adaptable in applications [1]. In this study, we use the Gumbel-Hougaard family copula to fix a failed system.

Methods

In this work, we have introduced fault tolerance factor, so that the failure of each unit for each subsystem, common cause failure as well as failure due to human error and robot, will be accompanied by this fault tolerance factor. This is to allow the system to recover from failure. The failure rate of each subsystem built for this system is constant. This system is susceptible to two types of failure: partial and complete failures. When a partial failure occurs, the system is not repaired; however, a total failure of the system can be fixed using one of the two repair facilities: Exponential or Copula. The system is observed at appropriate epoch by using regenerative point technique, supplementary variable techniques, and Gumbel-Hougaard family Copula. Though the system can be repaired using either an exponential distribution or at random (arbitrary), however, we believe that the interactive study of this should focus on increasing the system's availability as well as revenue generation. The various measures of reliability such as availability, reliability, mean time to system failure (MTTF), sensitivity for MTTF and cost analysis have been computed for various values of failure and repair rates. Maple 13 software has been used for computations.

Aim

In this present study, we analyze a series–parallel system composed of two subsystems with the following specifications: subsystem A consists of two similar units/components that are operated by Human, whereas subsystem B is made up of two similar units/components that are operated by Robot. The primary goals/aims of this work are to examine how fault tolerance factor will improve the availability and profit of the system under consideration and to determine the optimal repair strategies.

Originality

This research paper is the original work of authors. The references are well cited based on the importance of study. Nothing has been detached from any research paper or books.

Literature review and related work on human–robot collaboration

Many researchers have used the characteristics of the Gumbel-Hougaard family copula in the repair of failed systems and reported improved results, we can mention few among them, Yusuf et al. [2] used the copula repair technique to demonstrate the effectiveness of a multi-computer system with three subsystems in series configuration. Singh et al. [3] conducted a research via copula repair policy on the probabilistic evaluation of a CBT network system having four different subsystems in series. Tyagi et al. [4] demonstrated a copula analysis of a parallel system with fault coverage. Gulati et al. [5] have evaluated the performance analysis of complex repairable system in series connection under different failure and repair discipline via copula approach. Monika Gahlot et al. [6] have used Gumbel-Hougaard family Copula to study the efficiency of the repairable system in series connection under different types of failure and two types of repairs. Ismail et al. [7] have analyzed the performance evaluation of a hybrid series–parallel system with two human operators using Gumbel-Hougaard family copula. Chopra and Ram [8] used Gumbel-Hougaard Copula to present reliability measures for two dissimilar units in parallel.

Many researchers have investigated the performance of repairable systems under fault tolerance/coverage factor conditions. Among them are, Jain et al. [9] investigated a fault-tolerant machining system (FTMS) comprised of standbys and a skilled or trained repairman. Ram and Goyal [10] investigated the reliability of a flexible manufacturing system using a combination of copula and coverage approach and found that the mixed copula-coverage technique improves system's reliability. Jain and Gupta [11] proposed using multiple vacations and imperfect coverage to model the performance of a repairable machining system. Later in [12], Jain, Shekhar, and Rani investigate the optimal N-policy for MRP by incorporating noble features such as unreliable server, imperfect coverage, and reboot to make the model more versatile and closer to realistic situations. Wang et al. [13] investigated a system with a warm standby unit that provided imperfect coverage. They assumed that the coverage factors of active and standby units differed and compared the system to five repair distributions: exponential, gamma, uniform, normal, and deterministic. Ke and Liu [14] investigated a repairable system that was operating in a failure prone environment with a reboot delay, a repair facility, and imperfect coverage. Jain and Meena in [15] developed fault tolerant system performance models that include realistic features such as imperfect coverage, reboot, and server vacation. In addition, several researchers have submitted excellent works assessing the performance of repairable systems. Researchers such as,Raissi and Ebadi [16] studied computer simulation model for reliability estimation of a complex system. Pourhassan et al. [17] have suggested simulation approach/technique for reliability assessment of complex system under stochastic degradation and random shock. Pourhassan [18] evaluate reliability of power station subject fatal and non-fatal shocks.

Human errors can have a significant impact on the performance and economic results of production systems in a number of ways. Some of the ways in which human errors can affect production systems include reduced productivity, Increased rework, quality issues and safety concerns. It is important for companies to take steps to minimize the occurrence of human errors in their production systems in order to improve productivity, reduce costs, maintain quality, and ensure safety. This can involve providing proper training and supervision for employees, implementing quality control measures, and using technology and automation to reduce the likelihood of mistakes. The use of robots in manufacturing settings has become increasingly popular in recent years due to the advantages they offer. Robots can perform tasks faster and more accurately than humans, which can increase production output and reduce costs. Additionally, robots can work continuously without breaks or fatigue, which can further increase efficiency. Pairing humans with robots in a manufacturing setting can provide many benefits and is becoming increasingly necessary as technology continues to advance.

Human–robot collaboration is an emerging field that explores how robots and humans can work together effectively in various contexts, such as manufacturing, healthcare, and home assistance. The goal of this collaboration is to achieve shared objectives that require the complementary strengths of both robots and humans. There are many potential applications for human–robot collaboration, ranging from manufacturing and logistics to healthcare and service industries. For example, in manufacturing, robots can assist with assembly and quality control, while human operators can oversee the process and make decisions when necessary. In healthcare, robots can assist with patient care and rehabilitation, while human caregivers can focus on providing emotional support and building relationships with patients. Human–robot collaboration has the potential to revolutionize the way we work, by creating a more efficient, safe and productive work environment that leverages the strengths of both humans and robots.

Literature on human–robot collaboration in enhancing productivity and efficiency, as well as reduce the risk of accidents or injuries in the workplace are numerous. To cite few, Chen et al.'s [19, 20] proposed method has the potential to improve the efficiency and safety of human–robot collaboration tasks by enabling more accurate and reliable estimation of human arm stiffness and intention detection. Matheson et al. [21] highlight some of the challenges and considerations involved in implementing collaborative robotics in manufacturing settings, such as ensuring safety, training workers, and integrating robots into existing production processes. Chen et al. [19, 20] used a novel impedance mapping approach that involved two stages. The first stage involved identifying the human operator's impedance parameters by measuring the impedance of the operator's hand-arm system while performing a task. The second stage involved identifying the robot's impedance parameters by measuring the interaction force between the robot and the human operator during a collaborative task. The proposed impedance mapping strategy has several advantages. For example, it allows for real-time adjustment of the robot's impedance to match the operator's impedance, which can improve the accuracy and efficiency of the task. It also enables the robot to predict the operator's movements, which can improve the robot's ability to respond to the operator's actions. Overall, the proposed strategy has the potential to improve the effectiveness of human–robot collaboration in various settings, including manufacturing, healthcare, and service industries. Amarillo et al. [22] highlights the increasing demand for the integration of robotics in non-industrial settings where the environment is unpredictable and dynamic. The authors argue that in such environments, collaborative robots offer advantages over traditional industrial robots due to their ability to interact with humans and adapt to changing situations. Hiatt et al. [23] highlighted the importance of teamwork in human–robot collaboration, and how understanding each other can be challenging due to the differences between individuals. These differences can be mental, computational, or physical. To address this challenge, the authors suggested that developing explicit models of human teammates can help. These models can provide robots with a better understanding of how humans perceive and interpret their environment, as well as how they communicate and make decisions. By incorporating such models into the design of robots, they can become better team members and collaborators with humans. Mukherjee et al. [24] provide a taxonomy of levels of interaction between humans and robots that is based on the guidelines for autonomous vehicles created by SAE. The authors propose this taxonomy to standardize definitions in the field and reflect its evolving nature. Golda et al. [25] proposed a method for measuring the productivity gains associated with the substitution of human labor with industrial robots. The method involves decomposing labor productivity growth into three components: technical change, efficiency change, and factor substitution. By doing this, the authors were able to isolate the impact of robot adoption on productivity growth and quantify the extent to which it contributes to overall productivity growth. The method provides a useful framework for analyzing the impact of automation on productivity growth and can help policymakers and researchers better understand the implications of technological change for the labor market and economic growth.

One of the key advantages of human–robot collaboration is that it can help to reduce cycle times and increase productivity. For example, robots can perform repetitive or physically demanding tasks, while humans can focus on more complex or cognitive tasks that require decision-making or problem-solving skills.

In addition, human–robot collaboration can help to improve quality and reduce errors. Robots can be programmed to perform tasks with a high degree of precision, while humans can monitor the process and intervene if necessary. This can help to catch errors early on and prevent them from escalating into larger problems.

The literature above on human–robot collaboration suggests that this approach can lead to significant improvements in performance of any system. However, such literature failed to addressed Copula approach to reliability and performance analysis which allows for the modelling of the dependence between different components of a system. This approach takes into account the fact that the performance of a system is often influenced by the interaction of its various components, rather than just the individual performance of each component. By using Copula, it is possible to model the joint probability distribution of the system's components, taking into account their dependencies. This allows for a more accurate assessment of the system's overall performance and dependability, which is crucial in industries where system downtime or failure can have significant financial and reputational costs. The copula approach offers a valuable tool for analysing the performance of manufacturing systems, taking into account the complex interactions between their components. By using this approach, industries can make more informed decisions about the design, maintenance, and operation of their manufacturing systems, ultimately leading to more dependable and reliable operations.

Motivated by this fact, we are interested in the Copula approach to availability and cost analysis of a fault tolerance series–parallel system endowed with human-robotic operators in this present work. The impact of the fault tolerance factor in conjunction with Copula on the system availability and cost function has been captured.

Reliability models can be extremely helpful in evaluating the performance and effectiveness of human–robot collaboration systems. These models can help researchers and engineers understand how different factors, such as the capabilities of the robot and the skills of the human operator, affect the overall performance of the system. These reliability models can be an important tool for evaluating the strength, performance, and effectiveness of human–robot collaboration systems, and can help guide the development of new and more effective systems in the future.

The remainder of the paper is organized as follows: with Introduction Section serving as an introduction. Nomenclatures, assumptions, and model description are covered in Nomenclatures, assumptions and model description Section. Model formulation and solution Section is concerned with model formulation and solution, while Numerical simulations in specific cases Section provides numerical simulations in specific cases. Results and discussion Section gives the discussion of the results while the paper is concluded in Conclusion Section follow by references.

Nomenclatures, assumptions and model description

Nomenclatures

t: Scale of time.

s: Variables' Laplace transform.

$S_{i=0,1,2,3,4,5,6,7}$: Transitional states.

$\tau_1/\tau_2/\tau_r/\tau_h/\tau_{cc}$: Failure rate of subsystem 1/ Failure rate of subsystem B/ Failure rate due to robotic error/ Failure rate due to human error/ Common cause failure rate.

C: Fault tolerance factor.

$\rho(x)$: Rate of repair for the completely failed state.

$H_i(t), i = 0, 1, 2, 3, 4, 5, 6, 7$: The probability of the system being in state S_i at any given time t.

$\overline{H}_i(s)$: $H_i(t)$'s Laplace transform.

$H_i(x,t), i = 3,4,5,6,7$: The probability density function of the failed states of the system at any given time t, multiplied by the elapsed repair time x.

$\varnothing(x) = exp\left[x^\theta + \{log\rho(x)\}^\theta\right]^{\frac{1}{\theta}}$, where $\varnothing(x)$ is the joint probability of repair rate from completely failed state to perfect state, from Gumbel-Hougaard family Copula.

$\overline{S}(s) = \int_0^\infty \varnothing(x)e^{-sx-\int_0^\infty \varnothing(x)dx}dx$, where $\overline{S}(s)$ denotes the probability density function of the Laplace transformation of $\varnothing(x)$.

$E_p(t)$: Expected profit function during the time interval [0, t).

F_1, F_2: Revenue and service cost per unit time, respectively.

Assumptions

a. At first, the system is in a good state as both parallel subsystems are operational.
b. All failure rates are all constant and have exponential distribution. The exponential distribution is often used in reliability analysis to model systems with a constant failure rate, which means that the probability of failure of the system in a given time interval is constant, regardless of how long the system has been in operation. The exponential distribution is often used to model the time until failure of electronic components, machines, and systems that do not wear out over time, but instead have a constant failure rate. One advantage of using the exponential distribution in reliability analysis is that it is relatively simple to work with mathematically, and has a closed form solution for many important reliability metrics. Additionally, the exponential distribution is often a good approximation for the failure behavior of many systems, particularly those that have a constant failure rate.
c. Human and robotic failures, as well as common cause failure, can occur at any time, regardless of whether one or two units from both subsystems are operational.
d. Two repair facilities work together to repair the system in its completely failed state.
e. The repairs of completely failed states or units are modelled using Gumbel-Hougaard Family Copula or arbitrary exponential distribution.
f. The system functions as if it were new after being repaired.

Model description

This present study considered a series–parallel system with two subsystems, each with two similar units in parallel configurations. The first subsystem i.e., subsystem A is operated by human while the second subsystem i.e., subsystem B is operated by robot. If a fault occurs in the system, it recovers immediately using the fault tolerance factor C. However, if the system is unable to recover, then it enters a complete failure state and must be repaired back to its original state using an exponential distribution or Copula repair. Table 1 provides a brief description of the states, while Figs. 1 and 2 depicts all possible state transition for the model.

The interesting aspect of this system configuration is that subsystem B is operated by a robot, while subsystem A is operated by human operation. This indicates that the system uses both automation and human intervention in its operation. It is noted that the operation of the entire system is controlled by both human and robot unlike

Table 1 Description of states

States	Description
S_0	S_0 is the ideal state in which the two subsystems and their respective units work perfectly
S_1	In this state, the first unit of subsystem 1 has failed, but the other two units in subsystem 2 are working perfectly. The system is working
S_2	In this state, one unit has previously failed in subsystem 1, and the first unit from subsystem 2 unexpectedly failed, but the other units from both subsystems 1 and 2 are fully operational. The system is now operational
S_3	This state denotes total failure as a result of the failure of both units in subsystem 1
S_4	This state represents total failure as a result of the failure of both subsystem 2 units
S_5	This state denotes complete failure due to human operator error
S_6	This state also represents complete failure due to robot operator error
S_7	This state is a complete failure due to common cause failure

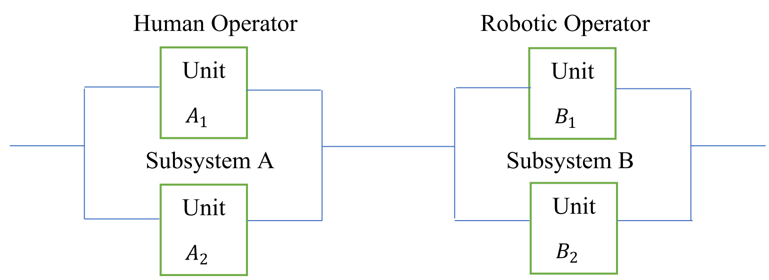

Fig. 1 Block diagram of the system

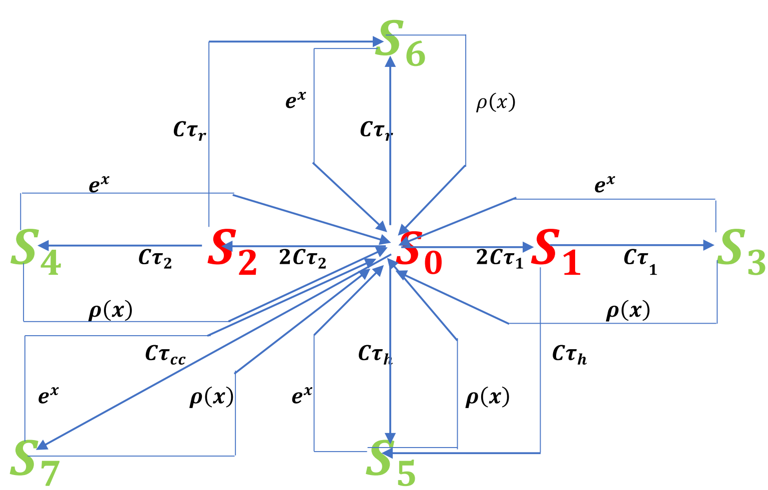

Fig. 2 Transition diagram of the system

in a normal series–parallel system, where the entire system is usually controlled by one or the other. This suggests that the system is designed to combine the strengths of both automation and human intervention, in order to achieve better performance or efficiency. The study aims to develop reliability models for a specific type of system consisting of two subsystems, A and B, with each subsystem having two identical units. Subsystem B is operated by a robot, while subsystem A is operated by a human. The overall operation of the system is controlled by both human and robot, unlike in a traditional series–parallel system.

To develop reliability models for this system, various factors such as the failure rate of each unit, the probability of the robot or human operator making errors, and the probability of the system functioning correctly under different operating conditions need to be considered. These factors can be used to create a mathematical model that predicts the system's reliability and helps to optimize its performance. The purpose of developing reliability models is to optimize the performance of the system. Reliability models help to predict the probability of the system functioning correctly over a specified period. By optimizing the system's reliability, we can improve its overall performance, reduce maintenance costs, and minimize downtime.

Model formulation and solution

Model formulation

For the model under consideration, one can derive the following set of difference-differential equations using elementary probability and continuity arguments as:

$$\begin{aligned}\left\{\frac{\partial}{\partial t}+2\tau_1C+2\tau_2C+\tau_rC+\tau_hC+\tau_{cc}C\right\}H_0(t)=&\int_0^\infty\left\{\rho(x)+e^x\right\}H_3(x,t)dx\\&+\int_0^\infty\left\{\rho(x)+e^x\right\}H_4(x,t)dx\\&+\int_0^\infty\left\{\rho(x)+e^x\right\}H_5(x,t)dx\\&+\int_0^\infty\left\{\rho(x)+e^x\right\}H_6(x,t)dx\\&+\int_0^\infty\left\{\rho(x)+e^x\right\}H_7(x,t)dx,\end{aligned}\tag{1}$$

$$\left\{\frac{\partial}{\partial t}+\tau_1C+\tau_hC\right\}H_1(x,t)=2\tau_1CH_0(t),\tag{2}$$

$$\left\{\frac{\partial}{\partial t}+\tau_2C+\tau_rC\right\}H_2(x,t)=2\tau_2CH_0(t),\tag{3}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(\rho(x)+e^x\right)\right\}H_3(x,t)=0,\tag{4}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(\rho(x)+e^x\right)\right\}H_4(x,t)=0,\tag{5}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(\rho(x)+e^x\right)\right\}H_5(x,t)=0,\tag{6}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}H_6(x,t)=0, \tag{7}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}H_7(x,t)=0. \tag{8}$$

Boundary conditions

$$H_3(0,t)=2{\tau_1}^2CH_0(t), \tag{9}$$

$$H_4(0,t)=2{\tau_2}^2CH_0(t), \tag{10}$$

$$H_5(0,t)=\tau_hC(1+2\tau_1C)H_0(t), \tag{11}$$

$$H_6(0,t)=\tau_rC(1+2\tau_2C)H_0(t), \tag{12}$$

$$H_7(0,t)=\tau_{cc}CH_0(t). \tag{13}$$

Initial condition

$$H_0(0)=1 \text{ and other state probabilities are zero at } t=0. \tag{14}$$

Model solution

Taking Laplace transforms of Eqs. (1, 2, 3, 4, 5, 6, 7 and 8) through Eq. (14), we obtain:

$$\begin{aligned}\left\{s+2\tau_1C+2\tau_2C+\tau_rC+\tau_hC+\tau_{cc}C\right\}\overline{H}_0(s)=&\int_0^\infty\left\{\rho(x)+e^{x}\right\}\overline{H_3}(x,s)dx\\&+\int_0^\infty\left\{\rho(x)+e^{x}\right\}\overline{H_4}(x,s)dx\\&+\int_0^\infty\left\{\rho(x)+e^{x}\right\}\overline{H_5}(x,s)dx\\&+\int_0^\infty\left\{\rho(x)+e^{x}\right\}\overline{H_6}(x,s)dx\\&+\int_0^\infty\left\{\rho(x)+e^{x}\right\}\overline{H_7}(x,s)dx,\end{aligned} \tag{15}$$

$$\{s+\tau_1C+\tau_hC\}\overline{H}_1(x,s)=2\tau_1C\overline{H}_0(s), \tag{16}$$

$$\{s+\tau_2C+\tau_rC\}\overline{H}_2(x,t)=2\tau_2C\overline{H}_0(s), \tag{17}$$

$$\left\{s+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}\overline{H_3}(x,s)=0, \tag{18}$$

$$\left\{s+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}\overline{H_4}(x,s)=0, \tag{19}$$

$$\left\{s+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}\overline{H_5}(x,s)=0, \tag{20}$$

$$\left\{s+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}\overline{H_6}(x,s)=0, \tag{21}$$

$$\left\{s+\frac{\partial}{\partial x}+\left(\rho(x)+e^{x}\right)\right\}\overline{H_7}(x,s)=0. \tag{22}$$

Boundary conditions

$$\overline{H}_3(0,s)=2{\tau_1}^2C\overline{H}_0(s), \tag{23}$$

$$\overline{H}_4(0,s)=2{\tau_2}^2C\overline{H}_0(s), \tag{24}$$

$$\overline{H}_5(0,s)=\tau_h C(1+2\tau_1 C)\overline{H}_0(s), \tag{25}$$

$$\overline{H}_6(0,s)=\tau_r C(1+2\tau_2 C)\overline{H}_0(s), \tag{26}$$

$$\overline{H}_7(0,s)=\tau_{cc} C\overline{H}_0(s). \tag{27}$$

We get the following equations i.e., (28, 29, 30, 31, 32, 33, 34 and 35) by solving (15, 16, 17, 18, 19, 20, 21 and 22) with the help of (23, 24, 25, 26 and 27).

$$\overline{H}_0(s)=\frac{1}{(s+2\tau_1C+2\tau_2C+\tau_rC+\tau_hC+\tau_{cc}C)-\overline{S}(s)\{2{\tau_1}^2C+2{\tau_2}^2C+\tau_hC(1+2\tau_1C)+\tau_rC(1+2\tau_2C)+\tau_{cc}C\}}, \tag{28}$$

$$\overline{H}_1(s)=\left\{\frac{2\tau_1C}{s+\tau_1C+\tau_hC}\right\}\overline{H}_0(s), \tag{29}$$

$$\overline{H}_2(s)=\left\{\frac{2\tau_2C}{s+\tau_2C+\tau_rC}\right\}\overline{H}_0(s), \tag{30}$$

$$\overline{H}_3(s)=\overline{H}_3(0,s)\left\{\left(\frac{1-\overline{S}(s)}{s}\right)\right\}, \tag{31}$$

$$\overline{H}_4(s)=\overline{H}_4(0,s)\left\{\left(\frac{1-\overline{S}(s)}{s}\right)\right\}, \tag{32}$$

$$\overline{H}_5(s)=\overline{H}_5(0,s)\left\{\left(\frac{1-\overline{S}(s)}{s}\right)\right\}, \tag{33}$$

$$\overline{H}_6(s)=\overline{H}_6(0,s)\left\{\left(\frac{1-\overline{S}(s)}{s}\right)\right\}, \tag{34}$$

$$\overline{H}_7(s)=\overline{H}_7(0,s)\left\{\left(\frac{1-\overline{S}(s)}{s}\right)\right\}, \tag{35}$$

The system's operational state availability is given by:

$$\overline{H}_{up}(s)=\overline{H}_0(s)+\overline{H}_1(s)+\overline{H}_2(s). \tag{36}$$

$$\overline{H}_{up}(s)=\left\{1+\left(\frac{2\tau_1 C}{s+\tau_1 C+\tau_h C}\right)+\left(\frac{2\tau_2 C}{s+\tau_2 C+\tau_r C}\right)\right\}\overline{H}_0(s). \tag{37}$$

Numerical simulations in specific cases

Availability analysis

System availability in the absence of both Copula and Fault tolerance factor

The expression for Laplace transforms of system availability in the absence of both Copula and Fault tolerance factor is presented by Eq. (38) below.

$$\overline{H}_{up}(s)=\left\{1+\left(\frac{2\tau_1}{s+\tau_1+\tau_h}\right)+\left(\frac{2\tau_2}{s+\tau_2+\tau_r}\right)\right\}\overline{H}_0(s), \tag{38}$$

where $\overline{H}_0(s)=\frac{1}{(s+2\tau_1+2\tau_2+\tau_r+\tau_h+\tau_{cc})-\frac{\rho}{s+\rho}\{2\tau_1{}^2+2\tau_2{}^2+\tau_h(1+2\tau_1)+\tau_r(1+2\tau_2)+\tau_{cc}\}}$.

Equation (39) is obtained by setting the failure rates in Eq. (38) to $\tau_1=0.01, \tau_2=0.02, \tau_h=0.03, \tau_r=0.04, \tau_{cc}=0.05$, and $\rho=1$, and performing the Laplace transform.

$$\begin{aligned}\mathrm{H_{up}(t)} = {} & 1.714285714e^{-0.04000000000t}-3.615384615e^{-0.06000000000t}\\ & +0.013433792772e^{-0.59000000000t}(215.8888603cosh(0.5397221507t)\\ & +199.sinh(0.5397221507t)).\end{aligned} \tag{39}$$

We obtain Table 2 and Fig. 3 for system availability in the absence of both Copula and Fault tolerance factor taking $t=0,5,10,15,20,25,30,35$, and so on.

System availability in the presence of Copula only

The expression in Eq. (40) represents system availability in the presence of a Copula while disregarding the Fault tolerance factor.

$$\overline{H}_{up}(s)=\left\{1+\left(\frac{2\tau_1}{s+\tau_1+\tau_h}\right)+\left(\frac{2\tau_2}{s+\tau_2+\tau_r}\right)\right\}\overline{H}_0(s), \tag{40}$$

where $\overline{H}_0(s)=\frac{1}{(s+2\tau_1+2\tau_2+\tau_r+\tau_h+\tau_{cc})-\frac{e}{s+e}\{2\tau_1{}^2+2\tau_2{}^2+\tau_h(1+2\tau_1)+\tau_r(1+2\tau_2)+\tau_{cc}\}}$.

Setting the failure rates to as $\tau_1=0.01, \tau_2=0.02, \tau_h=0.03, \tau_r=0.04, \tau_{cc}=0.05$, and $\rho=1, x=1$ *and* $\theta=1$, in Eq. (40) and taking Laplace transform yields Eq. (41).

Table 2 System availability in the absence of both Copula and Fault tolerance factor

Time (t)	Availability ($H_{up}(s)$)
0	1.0000
5	0.8936
10	0.8510
15	0.7822
20	0.7012
25	0.6171
30	0.5356
35	0.4597
40	0.3912
45	0.3306
50	0.2777
55	0.2321
60	0.1932
65	0.1603
70	0.1326
75	0.1094
80	0.0901

$$\begin{aligned} \mathrm{H_{up}(t)} = {} & 1.336895993e^{-0.040000000000t} - 6.68815889e^{-0.06000000000t} \\ & + 2.37691526\ 10^{-14}e^{-1.449150000t}\left(2.672058607\ 10^{14}cosh(1.394860668t)\right. \\ & \left.+2.634957381\ 10^{14}sinh(1.394860668t)\right). \end{aligned} \tag{41}$$

Taking $t = 0, 5, 10, 15, 20, 25, 30, 35,$ and so on, Table 3 and Fig. 4 show the system availability in the presence of Copula only.

System availability with fault tolerance factor ignoring Copula

The equation below represents system availability in the presence of Fault tolerance factor ignoring Copula.

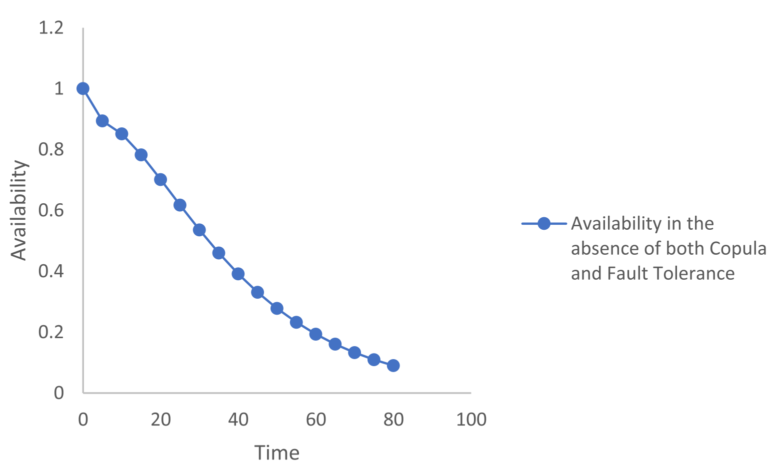

Fig. 3 System availability against time in the absence of both Copula and Fault tolerance factor

Table 3 System availability in the presence of Copula only

Time (t)	Availability ($H_{up}(s)$)
0	1.0000
5	0.9477
10	0.8905
15	0.8081
20	0.7158
25	0.6228
30	0.5345
35	0.4539
40	0.3822
45	0.3196
50	0.2657
55	0.2199
60	0.1813
65	0.1490
70	0.1221
75	0.0998
80	0.0814

$$\overline{H}_{up}(s) = \left\{1 + \left(\frac{2\tau_1 C}{s + \tau_1 C + \tau_h C}\right) + \left(\frac{2\tau_2 C}{s + \tau_2 C + \tau_r C}\right)\right\}\overline{H}_0(s) \tag{42}$$

where $\overline{H}_0(s) = \frac{1}{(s+2\tau_1 C+2\tau_2 C+\tau_r C+\tau_h C+\tau_{cc} C)-\frac{\rho}{s+\rho}\left\{2{\tau_1}^2 C+2{\tau_2}^2 C+\tau_h C(1+2\tau_1 C)+\tau_r C(1+2\tau_2 C)+\tau_{cc} C\right\}}$.

By substituting $\tau_1 = 0.01, \tau_2 = 0.02, \tau_h = 0.03, \tau_r = 0.04, \tau_{cc} = 0.05, \rho = 1$, and using $t = 0, 5, 10, 15, 20, 25, 30, 35$, and so on, and the Laplace transform, we obtain various numerical results for different values of the Fault tolerance factor, which are shown in Table 4 and Fig. 5 below.

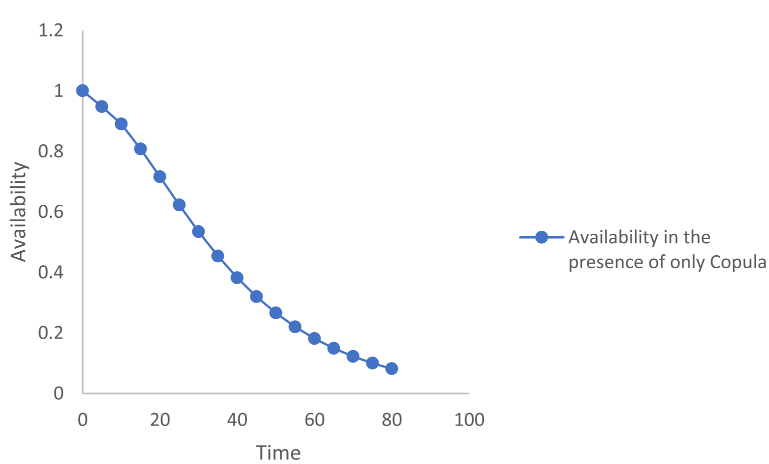

Fig. 4 System availability in the presence of Copula only

Table 4 System availability in the presence of Fault tolerance factor only

Availability $\overline{H}_{up}(s)$					
Time (t)	**$C = 0.1$**	**$C = 0.3$**	**$C = 0.5$**	**$C = 0.7$**	**$C = 0.9$**
0	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9886	0.9663	0.9448	0.9238	0.9035
10	0.9883	0.9617	0.9321	0.9004	0.8676
15	0.9873	0.9520	0.9079	0.8589	0.8079
20	0.9857	0.9379	0.8751	0.8059	0.7356
25	0.9834	0.9201	0.8363	0.7465	0.6588
30	0.9805	0.8994	0.7936	0.6845	0.5825
35	0.9770	0.8763	0.7486	0.6225	0.5098
40	0.9730	0.8513	0.7025	0.5622	0.4425
45	0.9684	0.8248	0.6563	0.5048	0.3815
50	0.9634	0.7973	0.6108	0.4510	0.3271

System availability in the presence of both Copula and Fault tolerance factor

Equation (43) denotes system availability when both Copula and Fault tolerance factor are used.

$$\overline{H}_{up}(s) = \left\{1 + \left(\frac{2\tau_1 C}{s + \tau_1 C + \tau_h C}\right) + \left(\frac{2\tau_2 C}{s + \tau_2 C + \tau_r C}\right)\right\}\overline{H}_0(s), \tag{43}$$

where$\overline{H}_0(s) = \frac{1}{(s+2\tau_1 C+2\tau_2 C+\tau_r C+\tau_h C+\tau_{cc} C)-\frac{e}{s+e}\{2\tau_1{}^2 C+2\tau_2{}^2 C+\tau_h C(1+2\tau_1 C)+\tau_r C(1+2\tau_2 C)+\tau_{cc} C\}}$.

Using $\tau_1 = 0.01, \tau_2 = 0.02, \tau_h = 0.03, \tau_r = 0.04, \tau_{cc} = 0.05$, and $\rho = 1, x = 1$ *and* $\theta = 1$, in Eq. (43), and letting $t = 0, 5, 10, 15, 20, 25, 30, 35$, and so on, then performing Laplace transform, we get various numerical outcomes for different values of Fault tolerance factor, which are presented in Table 5 and Fig. 6 given below.

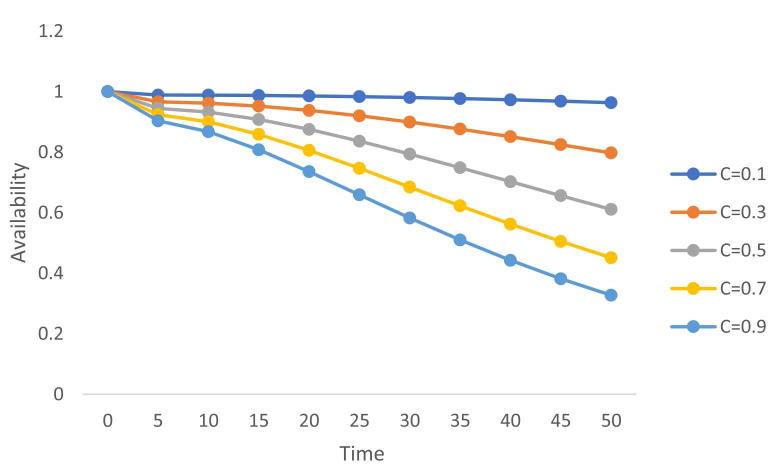

Fig. 5 System availability in the presence of Fault tolerance factor only

Table 5 System availability in the presence of both Copula and Fault tolerance factor

Availability $\overline{H}_{up}(s)$					
Time (t)	$C = 0.1$	$C = 0.3$	$C = 0.5$	$C = 0.7$	$C = 0.9$
0	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9959	0.9868	0.9766	0.9655	0.9038
10	0.9954	0.9806	0.9596	0.9342	0.9058
15	0.9943	0.9691	0.9311	0.8849	0.8343
20	0.9924	0.9533	0.8941	0.8249	0.7521
25	0.9899	0.9339	0.8516	0.7594	0.6672
30	0.9868	0.9116	0.8054	0.6922	0.5844
35	0.9831	0.8870	0.7572	0.6258	0.5069
40	0.9789	0.8605	0.7084	0.5620	0.4362
45	0.9741	0.8328	0.6598	0.5018	0.3729
50	0.9689	0.8040	0.6122	0.4460	0.3170

Cost function analysis

If the service facility is always open/available, the following formula can be used to calculate the expected/anticipated profit for the interval [0, t).

$$E_p(t) = F_1 \int_0^t H_{up}(t)dt - F_2 t. \quad (44)$$

Where F_1 and F_2 in the interval [0, t) represent revenue generated and service cost per unit time.

Cost function in the absence of both Copula and Fault tolerance factor

Using Eq. (38) in Eq. (44) with $\tau_1 = 0.01, \tau_2 = 0.02, \tau_h = 0.03, \tau_r = 0.04, \tau_{cc} = 0.05$, and $\rho = 1$, and taking the Laplace transform, we generate Eq. (45) for expected profit when both the Copula and Fault tolerance factor are ignored.

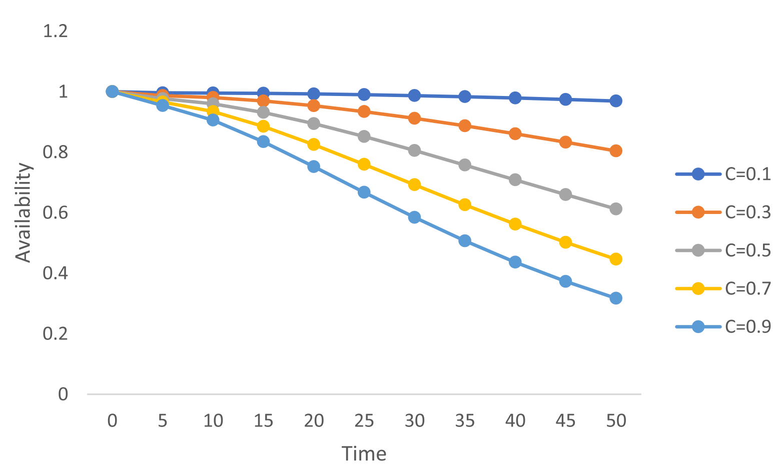

Fig. 6 System availability in the presence of both Copula and Fault tolerance factor

$$
\begin{aligned}
E_p(t) = {} & 60.25641025e^{-0.06000000000t} - 42.85714285e^{-0.04000000000t} \\
& - 55.44436162cosh(0.05027784930t) - 0.1004456201cosh(1.129722151t) \\
& + 55.44436162sinh(0.05027784930t) + 0.1004456201sinh(1.129722151t) \\
& + 38.14553984 - F_2t
\end{aligned} \tag{45}
$$

With $F_1 = 1,\ F_2 = 0.06, 0.05, 0.04, 0.03, 0.02, 0.01,$ and $t = 0, 1, 2, 3, 4, 5, 6, 7, , 8, 9, 10,$ we derive Table 6 and Fig. 7 from Eq. (45).

Cost function in the presence of Copula and in the absence of fault tolerance factor

Combining Eqs. (40) and (44) with $\tau_1 = 0.01, \tau_2 = 0.02, \tau_h = 0.03, \tau_r = 0.04, \tau_{cc} = 0.05,$ and $\rho = 1, x = 1$ *and* $\theta = 1$, yields Eq. (46), which is then transformed using Laplace transform.

$$
\begin{aligned}
E_p(t) = {} & 111.4692148e^{-0.06000000000t} - 33.42239982e^{-0.04000000000t} \\
& - 116.1768516cosh(0.05428933200t) - 0.01550389236cosh(2.844010668t) \\
& + 116.1768516sinh(0.05428933200t) + 0.01550389236sinh(2.844010668t) \\
& + 38.14554051 - F_2t
\end{aligned} \tag{46}
$$

Table 6 Expected profit in the absence of both Copula and Fault tolerance factor

Time	$E_p(t)$ $F_2 = 0.06$	$E_p(t)$ $F_2 = 0.05$	$E_p(t)$ $F_2 = 0.04$	$E_p(t)$ $F_2 = 0.03$	$E_p(t)$ $F_2 = 0.02$	$E_p(t)$ $F_2 = 0.01$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8981	0.9081	0.9181	0.9281	0.9381	0.9481
2	1.7553	1.7753	1.7953	1.8153	1.8353	1.8553
3	2.6000	2.6300	2.6600	2.6900	2.7200	2.7500
4	3.4398	3.4798	3.5198	3.5598	3.5998	3.6398
5	4.2756	4.3256	4.3756	4.4256	4.4756	4.5256
6	5.1065	5.1665	5.2265	5.2865	5.3465	5.4065
7	5.9310	6.0010	6.0710	6.1410	6.2110	6.2810
8	6.7476	6.8276	6.9076	6.9876	7.0676	7.1476
9	7.5548	7.6448	7.7348	7.8248	7.9148	8.0048
10	8.3516	8.4516	8.5516	8.6516	8.7516	8.8516

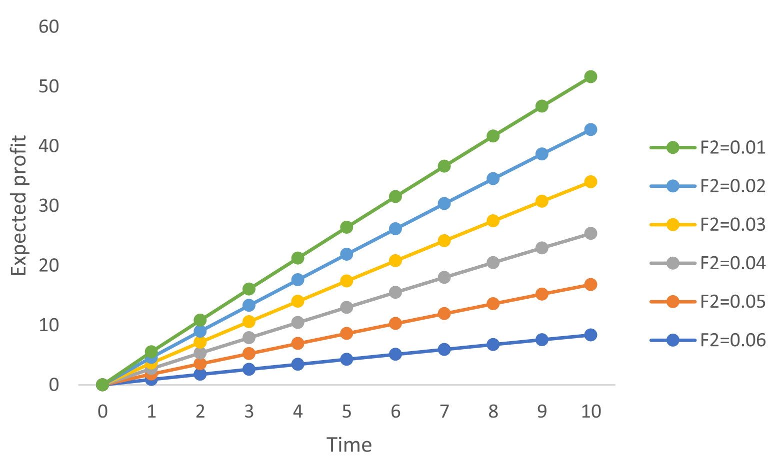

Fig. 7 Expected profit in the absence of both Copula and Fault tolerance factor

Using $F_1 = 1$, $F_2 = 0.06, 0.05, 0.04, 0.03, 0.02, 0.01$, and $t = 0, 1, 2, 3, 4, 5, 6, 7, , 8, 9, 10$, in Eq. (46), we obtain various expected profit results, which are shown in Table 7 and Fig. 8 below.

Cost function in the presence of fault tolerance factor and in the absence of Copula

a. *When C*=0.1

Equation (47) is obtained by combining Eqs. (44) and (42) with $\tau_1 = 0.01, \tau_2 = 0.02, \tau_h = 0.03, \tau_r = 0.04, \tau_{cc} = 0.05, \rho = 1$, fixing Fault tolerance factor C at 0.1 and the Laplace transform.

$$\begin{aligned} E_p(t) = \{&F_1 3415.807560e^{-0.006000000000t} - 273.326015e^{-0.004000000000t} \\ &+3511.075753 sinh(0.005807194000t) + 0.01189816817 sinh(2.844010668t) \\ &-3511.075753 cosh(0.005807194000t) - 0.01189816817 cosh(2.844010668t) \\ &+368.6061062\} - F_2 t. \end{aligned} \tag{47}$$

Table 7 Expected profit in the presence of Copula and in the absence of Fault tolerance factor

Time	$E_p(t)$ $F_2 = 0.06$	$E_p(t)$ $F_2 = 0.05$	$E_p(t)$ $F_2 = 0.04$	$E_p(t)$ $F_2 = 0.03$	$E_p(t)$ $F_2 = 0.02$	$E_p(t)$ $F_2 = 0.01$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9127	0.9227	0.9327	0.9427	0.9527	0.9627
2	1.8138	1.8338	1.8538	1.8738	1.8938	1.9138
3	2.7136	2.7436	2.7736	2.8036	2.8336	2.8636
4	3.6101	3.6501	3.6901	3.7301	3.7701	3.8101
5	4.5012	4.5512	4.6012	4.6512	4.7012	4.7512
6	5.3848	5.4448	5.5048	5.5648	5.6248	5.6848
7	6.2595	6.3295	6.3995	6.4695	6.5395	6.6095
8	7.1255	7.2055	7.2855	7.3655	7.4455	7.5255
9	7.6955	7.7855	7.8755	7.9655	8.0555	8.1455
10	8.6340	8.7340	8.8340	8.9340	9.0340	9.1340

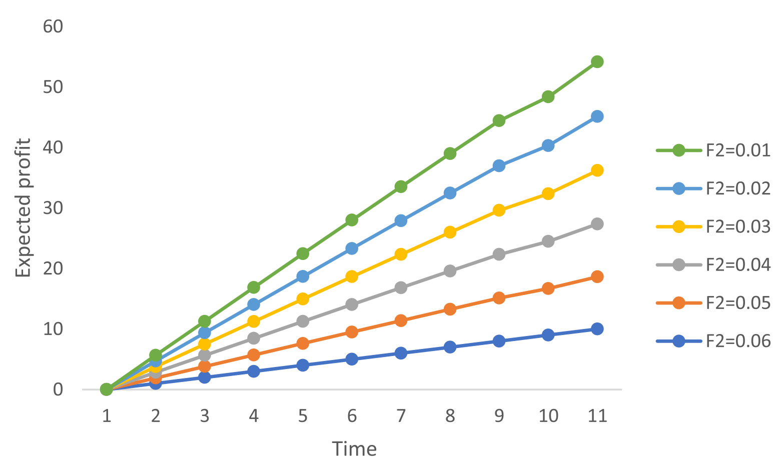

Fig. 8 Expected profit in the presence of Copula and in the absence of Fault tolerance factor

Taking $F_1 = 1,\ F_2 = 0.06, 0.05, 0.04, 0.03, 0.02, 0.01,$ and $t = 0, 1, 2, 3, 4, 5, 6, 7, , 8, 9, 10$ in Eq. (47), we obtain various expected profit results, which are presented in Table 8 and Fig. 9 below.

b. *When C*=0.5

Using the same parameters as in Eqs. (47) and (48) below represents the cost function in the presence of a Fault tolerance factor and in the absence of a Copula, but in this case C is fixed at 0.5.

$$\begin{aligned} E_p(t) &= 2226.9005848e^{-0.03000000000t} - 64.90066225e^{-0.02000000000t} \\ &- 236.7862248cosh(0.02724040350t) - 0.05537657164cosh(1.062759596t) \\ &+ 236.7862248sinh(0.02724040350t) + 0.05537657164sinh(1.062759596t) \\ &+ 74.84167877 - F_2t. \end{aligned} \tag{48}$$

Table 9 and Fig. 10 below display the numerical results for expected profit when C is set to 0.5.

Table 8 Expected profit in the presence of Fault tolerance factor and in the absence of Copula ($C = 0.1$)

Time	$E_p(t)$ $F_2 = 0.06$ $C = 0.1$	$E_p(t)$ $F_2 = 0.05$ $C = 0.1$	$E_p(t)$ $F_2 = 0.04$ $C = 0.1$	$E_p(t)$ $F_2 = 0.03$ $C = 0.1$	$E_p(t)$ $F_2 = 0.02$ $C = 0.1$	$E_p(t)$ $F_2 = 0.01$ $C = 0.1$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9356	0.9456	0.9556	0.9656	0.9756	0.9856
2	1.8666	1.8866	1.9066	1.9266	1.9466	1.9666
3	2.7959	2.8259	2.8859	2.8859	2.9159	2.9459
4	3.7247	3.7647	3.8047	3.8447	3.8847	3.9247
5	4.6533	4.7033	4.7533	4.8033	4.8533	4.9033
6	5.5819	5.6419	5.6419	5.7619	5.8219	5.8819
7	6.5104	6.5804	6.5804	6.7204	6.7904	6.8604
8	7.4389	7.5189	7.5189	7.6789	7.7589	7.8389
9	8.3673	8.4573	8.4573	8.6373	8.7273	8.8173
10	9.2957	9.3957	9.4957	9.5957	9.6957	9.7957

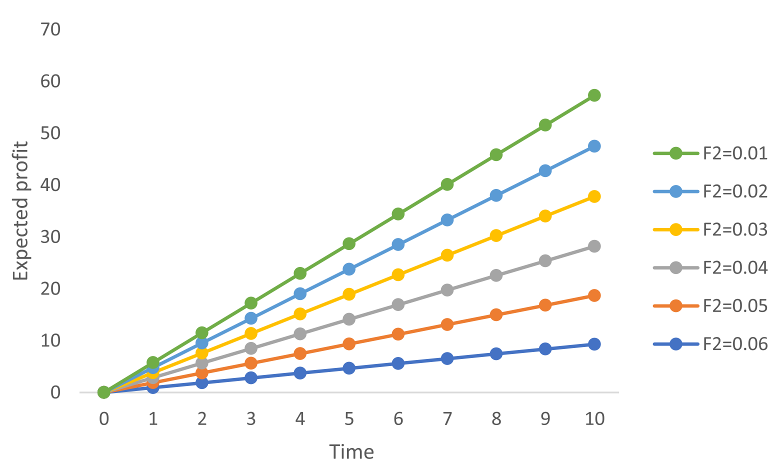

Fig. 9 Expected profit in the presence of Fault tolerance factor and in the absence of Copula ($C = 0.1$)

Table 9 Expected profit in the presence of Fault tolerance factor and in the absence of Copula ($C = 0.5$)

Time	$E_p(t)$ $F_2 = 0.06$ $C = 0.5$	$E_p(t)$ $F_2 = 0.05$ $C = 0.5$	$E_p(t)$ $F_2 = 0.04$ $C = 0.5$	$E_p(t)$ $F_2 = 0.03$ $C = 0.5$	$E_p(t)$ $F_2 = 0.02$ $C = 0.5$	$E_p(t)$ $F_2 = 0.01$ $C = 0.5$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9185	0.9285	0.9385	0.9485	0.9585	0.9685
2	1.8151	1.8351	1.8551	1.8751	1.8951	1.9151
3	2.7044	2.7344	2.7644	2.7944	2.8244	2.8544
4	3.5912	3.6312	3.6712	3.7112	3.7512	3.7912
5	4.4766	4.5266	4.5766	4.6266	4.6766	4.7266
6	5.3607	5.4207	5.4807	5.5407	5.6007	5.6607
7	6.2429	6.3129	6.3829	6.4529	6.5229	6.5929
8	7.1228	7.2028	7.2828	7.3628	7.4428	7.5228
9	8.0000	8.0900	8.1800	8.2700	8.3600	8.4500
10	8.8739	8.9739	9.0739	9.1739	9.2739	9.3739

iii. *When C*=0.9

Using the same parameters as in Eq. (48), Eq. (49) below represents the cost function in the presence of a Fault tolerance factor and in the absence of a Copula, but C is fixed at 0.9.

$$\begin{aligned} E_p(t) = {} & 74.07407407e^{-0.05400000000t} - 44.70413653e^{-0.03600000000t} \\ & - 71.49810129cosh(0.04598317780t) - 0.09223981135cosh(1.116016822t) \\ & + 71.49810129sinh(0.04598317780t) + 0.09223981135sinh(1.116016822t) \\ & + 42.22040356 - F_2t. \end{aligned} \tag{49}$$

Table 10 and Fig. 11 below show the numerical outcomes for expected profit when C is fixed at 0.9.

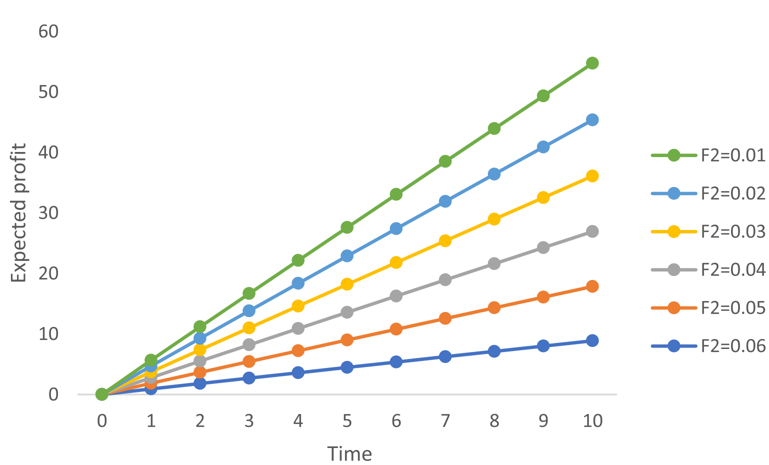

Fig. 10 Expected profit in the presence of Fault tolerance factor and in the absence of Copula ($C = 0.5$)

Table 10 Expected profit in the presence of Fault tolerance factor and in the absence of Copula ($C = 0.9$)

Time	$E_p(t)$ $F_2 = 0.06$ $C = 0.9$	$E_p(t)$ $F_2 = 0.05$ $C = 0.9$	$E_p(t)$ $F_2 = 0.04$ $C = 0.9$	$E_p(t)$ $F_2 = 0.03$ $C = 0.9$	$E_p(t)$ $F_2 = 0.02$ $C = 0.9$	$E_p(t)$ $F_2 = 0.01$ $C = 0.9$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9021	0.9121	0.9221	0.9321	0.9421	0.9521
2	1.7669	1.7869	1.8069	1.8269	1.8469	1.8669
3	2.6201	2.6501	2.6801	2.7101	2.7401	2.7701
4	3.4688	3.5088	3.5488	3.5888	3.6288	3.6688
5	4.3142	4.3642	4.4142	4.4642	4.5142	4.5642
6	5.1555	5.2155	5.2755	5.3355	5.3955	5.4555
7	5.9915	6.0615	6.1315	6.2015	6.2715	6.3415
8	6.8208	7.9008	6.9808	7.0608	7.1408	7.2208
9	7.6423	7.7323	7.8223	7.9123	8.0023	8.0923
10	8.4547	8.5547	8.6547	8.7547	8.8547	8.9547

System availability in the presence of both Copula and Fault tolerance factor

a. *When C*=0.1

Equations (43) and (44) were used to obtain Eq. (50) when $\tau_1 = 0.01, \tau_2 = 0.02, \tau_h = 0.03, \tau_r = 0.04, \tau_{cc} = 0.05$, and $\rho = 1, x = 1$ *and* $\theta = 1$, setting C to 0.1 and performing the Laplace transform.

With $F_1 = 1$, $F_2 = 0.06, 0.05, 0.04, 0.03, 0.02, 0.01$, and $t = 0, 1, 2, 3, 4, 5, 6, 7, , 8, 9, 10$ in Eq. (50), we obtain various expected profit results, which are presented in Table 7 and Fig. 12 below.

$$\begin{aligned} E_p(t) = {} & 4479.816547e^{-0.006000000000t} - 268.7975368e^{-0.004000000000t} \\ & - 4579.623437cosh(0.005851848000t) - 0.001629359126cosh(2.730448152t) \\ & + 4579.623437sinh(0.005851848000t) + 0.001629359126sinh(2.730448152t) \\ & + 368.6060564 - F_2 t. \end{aligned} \tag{50}$$

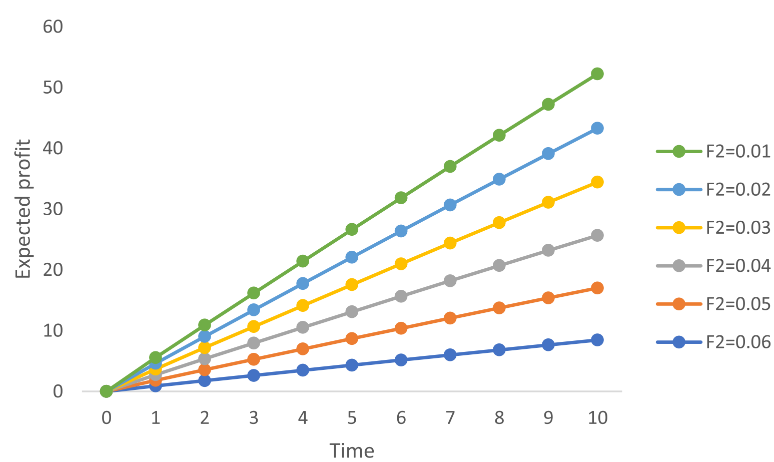

Fig. 11 Expected profit in the presence of Fault tolerance factor and in the absence of Copula ($C = 0.9$)

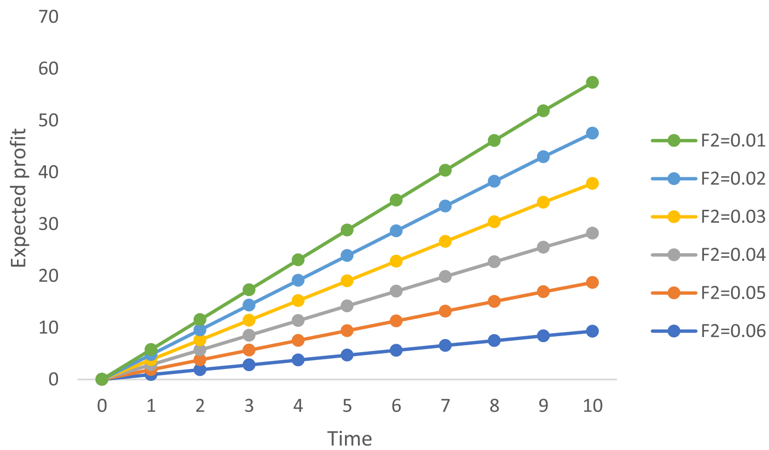

Fig. 12 Expected profit in the presence of both Copula and Fault tolerance factor ($C = 0.1$)

b. *When C*=0.5

Using the same procedures and parameters as in Eq. (50), we get Eq. (51) for expected profit when $C = 0.5$.

$$\begin{aligned} E_p(t) = {} & 385.0703073e^{-0.03000000000t} - 58.84088495e^{-0.03000000000t} \\ & + 401.0631200 sinh(0.02830755400t) + 0.007976854477 sinh(2.779992446t) \\ & - 401.0631200 coshh(0.02830755400t) - 0.007976854477 cosh(2.779992446t) \\ & + 74.84167445 - F_2 t. \end{aligned} \tag{51}$$

As in Eq. (50), we generate Table 12 and Fig. 13 when $C = 0.5$, for expected profit.

iii. *When C*=0.9

Using the same procedures and parameters as in Eq. (51), we obtain Eq. (52) for expected profit when $C = 0.9$.

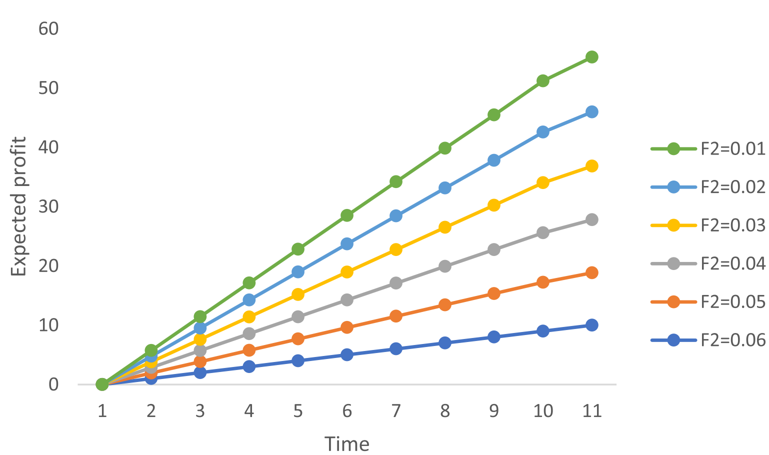

Fig. 13 Expected profit in the presence of both Copula and Fault tolerance factor ($C = 0.5$)

$$\begin{aligned}E_p(t) &= 135.35555059e^{-0.05400000000t} - 36.14675131e^{-0.03600000000t} \\ &+ 141.4151220sinh(0.04927462700t) + 0.01403621558sinh(2.831025373t) \\ &- 141.4151220cosh(0.04927462700t) - 0.01403621558cosh(2.831025373t) \\ &+ 42.22040363 - F_2t.\end{aligned} \tag{52}$$

Table 13 and Fig. 14 are generated when $C = 0.9$ for expected profit, as in Eq. (51).

Results and discussion

In order to gain a clear understanding of this study, this section discusses numerical results with reference to availability and cost function for the established model. Tables 2, 3, 4 and 5 and their corresponding Figs. 3, 4, 5 and 6 depict the system availability over time, when the repair rate follows an exponential distribution and the fault tolerance factor is not used, when the repair rate follows a copula distribution and the fault tolerance factor is not used, when the repair rate follows an exponential distribution and the fault tolerance factor is used, and when the repair rate follows a copula distribution and the fault tolerance factor is also used, respectively. Tables 2, 3, 4 and 5 and their corresponding Figs. 3, 4, 5 and 6 show that the availability of the system decreases over time in all situations. However, it is clear from these tables and figures that the system's availability is higher in all cases of fault tolerance factor, in particular, Table 4 and Fig. 5 are used, in which the repair rate follows a copula distribution and the fault tolerance factor is invoked. This analysis suggests that, the best technique for improving system availability is when the rate of repair follows a Copula distribution and the fault tolerance factor is used. It is also interesting to note that as the value of the fault tolerance factor increases, the system availability decreases, Tables 4 and 5, as well as their associated Figs. 6 and 7, illustrate this. This analysis highlights the risk of allowing the system to always recover.

Tables 6, 7, 8, 9, 10, 11, 12 and 13 and their related Figs. 7, 8, 9, 10, 11, 12, 13 and 14 show the value of expected profit over time for various service cost values when generated revenue is fixed at 1. Table 6 and Fig. 7 show the expected profit when repair rate follows an exponential distribution and the fault tolerance factor is not used. The

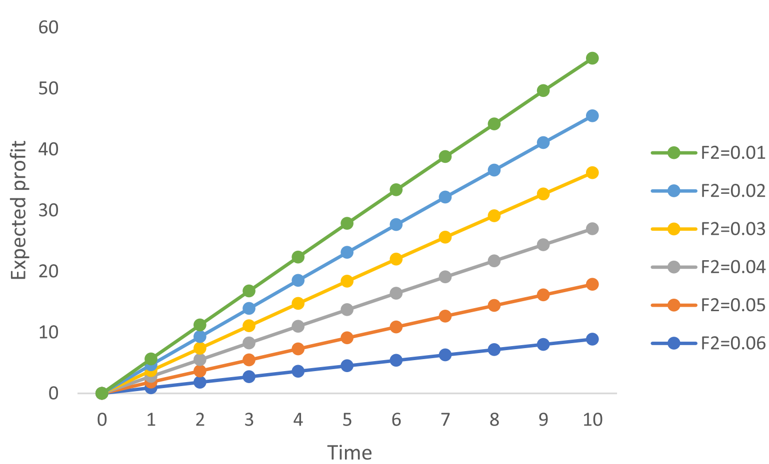

Fig. 14 Expected profit in the presence of both Copula and Fault tolerance factor ($C = 0.9$)

Table 11 Expected profit in the presence of both Copula and Fault tolerance factor ($C = 0.1$)

Time	$E_p(t)$ $F_2 = 0.06$ $C = 0.1$	$E_p(t)$ $F_2 = 0.05$ $C = 0.1$	$E_p(t)$ $F_2 = 0.04$ $C = 0.1$	$E_p(t)$ $F_2 = 0.03$ $C = 0.1$	$E_p(t)$ $F_2 = 0.02$ $C = 0.1$	$E_p(t)$ $F_2 = 0.01$ $C = 0.1$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9371	0.9471	0.9571	0.9671	0.9771	0.9871
2	1.8730	1.8930	1.9130	1.9330	1.9530	1.9730
3	2.8089	2.8389	2.8689	2.8989	2.9289	2.9589
4	3.7447	3.7847	3.8247	3.8647	3.9047	3.9447
5	4.6806	4.7306	4.7806	4.8306	4.8806	4.9306
6	5.6164	5.6764	5.7364	5.7964	5.8564	5.9164
7	6.5523	6.6223	6.6923	6.7623	6.8323	6.9023
8	7.4881	7.5681	7.6481	7.7281	7.8081	7.8881
9	8.4161	8.5061	8.5961	8.6861	8.7761	8.8661
10	9.3061	9.4061	9.5061	9.6061	9.7061	9.8061

Table 12 Expected profit in the presence of both Copula and Fault tolerance factor ($C = 0.5$)

Time	$E_p(t)$ $F_2 = 0.06$ $C = 0.5$	$E_p(t)$ $F_2 = 0.05$ $C = 0.5$	$E_p(t)$ $F_2 = 0.04$ $C = 0.5$	$E_p(t)$ $F_2 = 0.03$ $C = 0.5$	$E_p(t)$ $F_2 = 0.02$ $C = 0.5$	$E_p(t)$ $F_2 = 0.01$ $C = 0.5$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9260	0.9360	0.9460	0.9560	0.9660	0.9760
2	1.8458	1.8658	1.8858	1.9058	1.9258	1.9458
3	2.7653	2.7953	2.8253	2.8553	2.8853	2.9153
4	3.6841	3.7241	3.7641	3.8041	3.8441	3.8841
5	4.6016	4.6516	4.7016	4.7516	4.8016	4.8516
6	5.5171	5.5771	5.6371	5.6971	5.7571	5.8171
7	6.4297	6.4997	6.5697	6.6397	6.7097	6.7797
8	7.3317	7.4117	7.4917	7.5717	7.6517	7.7317
9	8.2417	8.3417	8.4417	8.5417	8.6417	8.7417
10	8.8417	8.9417	9.0417	9.1417	9.2417	9.3417

Table 13 Expected profit in the presence of both Copula and Fault tolerance factor ($C = 0.9$)

Time	$E_p(t)$ $F_2 = 0.06$ $C = 0.9$	$E_p(t)$ $F_2 = 0.05$ $C = 0.9$	$E_p(t)$ $F_2 = 0.04$ $C = 0.9$	$E_p(t)$ $F_2 = 0.03$ $C = 0.9$	$E_p(t)$ $F_2 = 0.02$ $C = 0.9$	$E_p(t)$ $F_2 = 0.01$ $C = 0.9$
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9153	0.9253	0.9353	0.9453	0.9553	0.9653
2	1.8201	1.8401	1.8601	1.8801	1.9001	1.9201
3	2.7237	2.7537	2.7837	2.8137	2.8437	2.8737
4	3.6247	3.6647	3.7047	3.7447	3.7847	3.8247
5	4.5213	4.5713	4.6213	4.6713	4.7213	4.7713
6	5.4118	5.4718	5.5318	5.5918	5.6518	5.7118
7	6.2944	6.3644	6.4344	6.5044	6.5744	6.6444
8	7.1604	7.2404	7.3204	7.4004	7.4804	7.5604
9	8.0204	8.1204	8.2204	8.3204	8.4204	8.5204
10	8.8804	8.9904	9.1004	9.2104	9.3204	9.4304

expected profit from the system when the repair rate follows a copula distribution and the fault tolerance factor is not used is depicted in Table 7 and Fig. 8. Tables 8, 9 and 10 and Figs. 9, 10 and 11 give the value of profit at different values of fault tolerance factor i.e., $C = 0.1$, $C = 0.5$,$C = 0.9$, respectively and the rate of repair follows an exponential distribution. While the expected profit when the rate of repair follows a Copula distribution and the fault tolerance factor is invoked is presented in Tables 11, 12 and 13 and their corresponding Figs. 12, 13, and 14. According to these tables and figures, namely Tables 6, 7, 8, 9, 10, 11, 12 and 13 and Figs. 7, 8, 9, 10, 11, 12, 13 and 14, the service cost is inversely proportional to the expected profit. In other words, when the service cost is low, the expected profit is highest and when the service cost is high, the expected profit is lowest. Many industries exist to make profit, and none of them will thrive if the cost of maintaining them is too high. The same scenario is observed in cost/benefit function, where availability appears to be higher in all cases of fault tolerance factor. As a result, in order to maximize profit, service costs should be kept under control, and the tolerance factor should be used.

Conclusions

Concerning the disparity between input and output for industrial systems, any process industry's growth is determined by the availability of its assets, maintenance strategy/technique, and revenue generated. As a result, in order to get the most out of operating systems, these factors must be meticulously maintained so that the rate/level of failure and repair is kept to a minimum. In this manner, the expected revenue generated by the system can be optimized.

Given the foregoing, the authors in this study, analyzed the availability and cost–benefit of a series–parallel system consisting of two subsystems operated by human and robot using the features of the Gumbel-Hougaard family Copula in conjunction with fault tolerance factor. The performance of the model under consideration was investigated using three different approaches, including Copula, Fault tolerance factor, and Copula-Fault tolerance factor, to determine how availability and expected profit can be improved. The basic expressions for system availability and the cost function are obtained in the following order. Availability in the absence of both Copula and Fault tolerance factor, Availability in the presence of only Copula, Availability in the presence of only Fault tolerance factor, Cost function in the absence of both Copula and Fault tolerance factor, Cost function in the presence of only Copula, Cost function in the presence of only Fault tolerance factor, Cost function in the presence of both Copula and Fault tolerance factor. These expressions have been validated numerically and are presented in tables and figures. Based on the numerical results obtained for a specific case in Tables 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13 and Figs. 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 and 14, it is clear that the optimum system availability and benefit can be obtained when the entire system is periodically repaired by Copula and the Fault tolerance factor has been invoked. It is common knowledge that system failure will reduce production performance and may even result in a tragedy; however, the introduction/addition of a Fault tolerance factor makes a significant contribution in improving system availability as well as expected profit. Hence, in order to improve system availability and revenue generation, the system should be repaired using Copula in conjunction with Fault tolerance factor. This study is extremely beneficial to system engineers and maintenance managers to accept multi-dimension repair in the form of Copula repair with fault tolerance factor. Also, when modified, the formal model developed in this work will allow system engineers to design more complex\sophisticated systems and results in a more competitive, efficient, and value-added production process by responding more quickly to technical or human errors to avoid system damage while increasing workplace safety and associated costs.

This study can be expanded by adding more units to each subsystem and implementing online and offline preventive maintenance. This subject will be explored more in our future work.

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Authors' contributions
The study's conception and design were contributed to by the authors. Material preparation, analysis and discussion of results were done by the named authors. All authors have read and approved the manuscript.

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Availability of data and materials
The authors declare that all the data and materials supporting the findings of this study are available within the article.

Declarations

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Competing interests
The authors whose names are written on this manuscript certify that they have NO affiliations with or involvement in any organization or entity with any financial interest in the subject matter or materials discussed in this manuscript.

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