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Assessing the accuracy of empirical decline curve techniques for forecasting production in unconventional reservoirs: a case study of Haynesville, Marcellus, and Marcellus Upper Shale

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Abstract

Decline curve analysis (DCA) is a widely used method to estimate the production performance and ultimate recovery of conventional and unconventional reservoirs. Due to the unconventional nature of shale wells, traditional decline curve methods are not ideal for analyzing their production decline behavior. In response, different empirical methods have been developed and used which rely on different mathematical and statistical approaches and can result in varying forecasts.

This study compares fourteen decline curve methods, along with the traditional Arps method, in terms of their ability to match production history, sensitivity to data size, effect of flow type, production forecast, and ultimate recovery estimation.

The methods were tested on three wells from Haynesville Shale (Lorikeet Field), Marcellus Shale (Ostrich Field), and Marcellus-Upper Shale (Penguin Field) respectively. The study concludes that each method may be useful in different cases, and engineers should choose the method that best models their wells based on their specific characteristics and circumstances. Recommendations were also provided for an effective evaluation of uncertainty and application of DCA. The primary objective of this study is to improve the accuracy and reliability of DCA predictions for different reservoir types with different declining modes.

Keywords: Decline curve analysis production performance, Ultimate recovery, Oil and gas reservoirs, unconventional reservoirs, Shale gas, Uncertainty evaluation, Haynesville shale, Marcellus shale, Prediction tool

Introduction

Arps' Decline Analysis equations are a reliable tool for estimating the ultimate recovery of conventional oil and gas wells by fitting and extrapolating production rate-time plots during Boundary Dominated Flow (BDF) to abandonment. However, these equations are not suitable for estimating reserves in shale wells because shale wells often exhibit long transient flow regimes due to their low matrix permeability. Traditional decline methods tend to overestimate reserves when applied to the transient flow regimes of shale wells. To address this issue, fifteen empirical equations from the literature were used, which produce different fitting and predicting performance. These methods are classified into methods that considers the flow behavior of shale gas and methods that do not considers the flow behavior of shale gas. The paper summarizes and critiques these methods, addressing their major differences, assumptions, and conditions.

In this study, a Microsoft excel using Visual Basic Application (VBA) was developed to automatically run and compare the methods on reliable field data. The program uses non-linear regression to find the best match between the production data and the equations, and the least squares method was employed, which involved minimizing the sum of the squared differences between the actual values and the calculated values of the field data. Also, the study proposes valuable conclusions and recommendations aimed at improving the understanding of uncertainties and enhancing the effective use of DCA.

Challenges related to analyzing the producing shale gas wells

To produce shale gas reservoirs, horizontal wells are drilled with multiple stages of fracture stimulation, resulting in a stimulated reservoir volume (SRV). The size of the SRV depends on the length of the horizontal well and the number of stages. However, this creates a complex system consisting of interconnected natural fractures, hydraulic fractures, and ultra-low permeability matrix, which resulting in various flow regimes. The decline trends of shale gas wells are impacted by the different flow regimes that may occur as shown in Fig. 1.

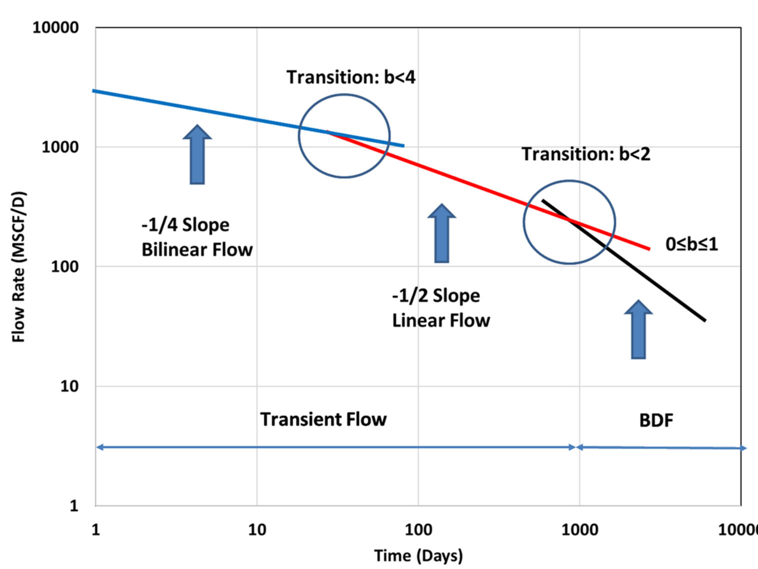

Fig. 1 Identifying the different flow regimes based on the slope value on the log–log plots

The various flow regimes that can occur during the production of shale gas wells can affect the decline trends, and these cases are as follows [1, 2]:

- *Linear flow*: which can dominate during the entire life of the well, occurs perpendicular to the hydraulic fractures from the matrix. This type of transient linear flow can be identified by plotting flow rate versus time on a Log–Log plot. The resulting plot exhibits a -1/2 slope if there are no natural fractures in the reservoir.
- *Linear-BDF*" refers to a combination of two flow regimes in shale gas wells. The initial flow regime is transient linear flow, which is followed by a boundary dominated flow (BDF). This transition is usually related to reaching the boundaries of the stimulated reservoir volume (SRV). On a log–log plot, this flow regime can be identified when a deviation from the -1/2 slope of the transient linear flow regime begins to appear.
- *Bilinear-Linear flow*: refers to a flow regime where the initial assumption is bilinear flow, which is followed by another linear flow. This type of flow is typically associated with the presence of natural fractures and lasts for a brief period during the early stages of production. In bilinear-linear flow, the fluid moves linearly from the fractures to the well and simultaneously from the matrix linearly to the fractures. On a Log–Log plot, it is identified by a slope of -1/4, followed by a slope of -1/2 that signifies the bilinear flow.
- *Bilinear-linear-BDF flow*: The Log–Log plot shows a deviation from the -1/2 slope once again when the flow reaches the boundaries of the stimulated reservoir volume (SRV). This flow regime starts with bilinear flow, followed by linear flow, and then transitions to boundary dominated flow (BDF) as the SRV is depleted

Methods

Empirical methods are classified into two categories:

- Methods that neglect the flow behavior of shale gas.
- Methods that take into account the flow behavior of shale gas

Methods that neglect the flow behavior of shale gas

Arps model

The Arps model comprises exponential, hyperbolic, and harmonic methods [4] that are characterized by the decline exponent b, which typically falls between 0 and 1 as demonstrated in Eq. 1 through 3. This model is applicable when the well is in BDF, and physical properties and BHP are nearly constant. However, b is always greater than 1 in shale gas reservoirs, rendering the super-hyperbolic model (where $b > 1$) necessary. The super-hyperbolic model employs the same expression as Eq. 2 but has different ranges for b.

Exponential Decline:

$$q_t = q_i exp(-D_i t), \tag{1}$$

Hyperbolic Decline:

$$q_t = \frac{q_i}{(1 + bD_i t)^{1/b}}, \tag{2}$$

Harmonic Decline:

$$q_t = \frac{q_i}{(1 + D_i t)}, \tag{3}$$

where; q_t = gas flow rate at time t (Mscf/day), q_i = initial gas flow rate (Mscf/day), t = time (Day), Di = initial decline rate (Day^{-1}), b = Arps' decline-curve exponent.

Matthews-Lefkovits Model (MLM)

Matthews and Lefkovits (1956) [28] (MLM) studied the performance of depletion-type reservoirs in the stage where gravity is the primary driving force due to low gas pressure. They conducted both theoretical analysis and experiments using scaled methods. They also developed a straightforward theoretical model that explains the scaled model experiments and matches some field data as shown in Eq. 4 [28],

$$q = q_i \cdot \frac{1}{(a_{MLM} \cdot t/n_{MLM} + 1)^{n_{MLM}}}, \tag{4}$$

where; a_{MLM} is a constant; and n_{MLM} is a fitting coefficient.

The similarity between the model and Arps's hyperbolic relationship is significant. It can be said that they are almost identical, as n_{MLM} approaches $1/b_A$ and a_{MLM} approaches $D_{A.}$

Generalized Weng's prediction model

Weng's prediction model is a mathematical model that is utilized to depict the increase and decrease of phenomena. The model was initially proposed by [39], and it has been further developed and studied by other researchers, including [43] as expressed in Eq. 5.

$$q = q_{\mathrm{i}} \cdot t^{a_{\mathrm{GW}}} \exp(-t/b_{\mathrm{GW}}), \tag{5}$$

where; a_{GW} and b_{GW} are fitting coefficients.

Extended КОПЫТОВ (K) Model

The КОПЫТОВ model is originally expressed as in Eq. 6 [24]. However, this model cannot be used directly because as t approaches 0, NP approaches negative infinity instead of 0. To address this issue, the extended КОПЫТОВ model (K) was modified to better fit actual production data [24] and is expressed as Eq. 7. The cumulative production can be obtained by from Eq. 8.

$$G_{\mathrm{P}} = a_{\mathrm{K}} - \frac{b_{\mathrm{K}}}{t}, \tag{6}$$

$$q = q_{\mathrm{i}} \cdot \left(\frac{b_{\mathrm{K}}}{b_{\mathrm{K}} + t}\right)^2 \tag{7}$$

$$G_P = \frac{a_K}{b_K} - \frac{a_K}{t + b_K} \tag{8}$$

where; a_K and b_K are fitting coefficients.

Stretched Exponential Decline Production Model (SEDP)

Kisslinger utilized the stretched exponential function to model aftershock decay rates in 1993 [20]. This stretched decay can be viewed as a combination of pure exponential decay, as discussed [17]. Valkó and Lee applied this concept to design their novel model for predicting the decline rates of production data from unconventional reservoirs, as presented in [21, 34, 35]. Despite bearing similarity to PLE, this model does not rely on single parameter interpretation and instead uses gamma functions. Parameters (n) and (τ) represent multiple exponential declines summations instead of being single parameters. Moreover, EUR is no longer an infinite but bounded value and is independent of time or rate, as noted by [47]. This model can fit both transient and BDF and is empirical, lacking any physical basis. It was derived by analyzing production data from over 12,800 wells of the Barnett Shale Play and expressed through a differential equation [3]. Eqs. 9, 10 and 11 demonstrate the mathematical methods derived from this model.

$$q = q_i \cdot \exp\left[-(t/\tau_{\mathrm{SEDP}})^{n_{\mathrm{SEDP}}}\right], \tag{9}$$

$$\mathrm{G_P} = \mathrm{q_i}\left(\frac{\tau_{\mathrm{SEDP}}}{n_{\mathrm{SEDP}}}\right)\left\{\Gamma\left(\frac{1}{n_{\mathrm{SEDP}}}\right) - \Gamma\left[\frac{1}{n_{\mathrm{SEDP}}}, \left(\frac{\mathrm{t}}{\tau_{\mathrm{SEDP}}}\right)^{n_{\mathrm{SEDP}}}\right]\right\}, \tag{10}$$

$$\mathrm{EUR} = \frac{qi^{\tau_{\mathrm{SEDP}}}}{n_{\mathrm{SEDP}}}\tau_{\mathrm{SEDP}}\left[\frac{1}{n_{\mathrm{SEDP}}}\right] \tag{11}$$

where; n_{SEDP} is the model parameter (dimensionless), τ_{SEDP}=model parameter (day), $\Gamma\left(\frac{1}{n}\right)$ is gamma function and $\Gamma\left[\frac{1}{n}, \left(\frac{t}{\tau}\right)^n\right]$ is the incomplete gamma function.

It is worth noting that the model takes into account reservoir heterogeneity, as discussed in [12]. Accurately estimating the parameters (n) and (τ) requires a significant amount of production data, typically over 36 months, as emphasized in [27].

Weibull's model

A statistical distribution model known as the Weibull distribution was proposed by Weibull in 1951 [38] and further studied by Tabatabai et al. in 2005 [33] and Yu in 1994. This model has been extensively utilized for life testing purposes. In the petroleum industry, a modified version of this model, as demonstrated in Eq. 12, is employed for predicting well production [33, 41]:

$$q = q_\mathrm{i} \cdot \exp\left(-\frac{t^{n_{\mathrm{WB}}}}{a_{\mathrm{WB}}}\right), \tag{12}$$

where; a_{WB} and n_{WB} are fitting coefficients.

Weng and Weibull's methods depend on the statistical behaviors as they are considered probability density functions. They are sensitive to the data sample and the coverage range of that sample. Therefore, the production data size available for fitting by these two methods could be essential before forecasting the production.

Logistic Growth Model (LGM)

The logistic methods operate on the principle that growth can occur only up to a certain limit, referred to as the carrying capacity. LGMs are employed for modeling population growth and were initially adapted for use in the petroleum industry by Hubbert in 1956. Hubbert's model was utilized for predicting the production of a field or region. Clark et al. [7] developed a new LGM that focuses on the production of an individual well. This model was derived from another LGM that methods liver regrowth using hyperbolic functions. The rate and cumulative equations for this model are as in Eq. 13.

$$q = \frac{K \cdot n_{\mathrm{LGM}} \cdot a_{\mathrm{LGM}} \cdot t^{n_{\mathrm{LGM}}-1}}{(a_{\mathrm{LGM}} + t^{n_{\mathrm{LGM}}})^2}, \tag{13}$$

where; a_{LGM} is a constant; and n_{LGM} is a hyperbolic exponent.

The two regression variables, (a_{LGM}) and (n_{LGM}), significantly influence the shape and direction of the decline curve, either upward or downward. K, which represents the carrying capacity or the maximum achievable limit, can be the EUR or the initial maximum rate, as indicated in Eq. 14 [8].

$$q = q_{\mathrm{i}} \cdot \frac{n_{\mathrm{LGM}} - a_{\mathrm{LGM}} \cdot t^{n_{LGM}-1}}{(a_{\mathrm{LGM}} + t^{n_{LGM}})^2}, \tag{14}$$

Methods that take into account the flow behavior of shale gas

Hsieh model

Hsieh et al. [13] proposed a Darcy relationship that is time-dependent to account for unique reservoir properties of a well that vary with pressure over time. Equation 15 of this model incorporates three fitting parameters that regulate the well's declining behavior within a specific geographic region. This model offers simplicity and versatility as it can analyze various decline scenarios without prior knowledge of the decline type. It can be viewed as a modification of Arps's model, aimed at making the decline more comprehensive.

$$q = q_i \cdot t^{-(n_H + m_H.t)}, \tag{15}$$

where; m_H is a time-dependent hysteresis decline exponent, and n_H is a decline exponent.

Power-law equation (PLE) Model

According to [15, 16], the use of a hyperbolic relationship in tight gas reservoirs is not a reliable method for estimating reserves through extrapolation. Instead, they proposed a more general relationship, which is deemed to be more accurate. They introduced a new approach called the "Power-Low loss-ratio" rate decline model, and its details can be found in Eqs. 16 and 17.

$$q(t) = q_i e^{\left[-D_\infty t - \frac{D1}{n_{PLE}} t^{n_{PLE}}\right]} \tag{16}$$

when $D_\infty \approx 0$,

$$q(t) = q_i e^{\left[-\frac{D1}{n_{PLE}} t^{n_{PLE}}\right]} \tag{17}$$

where; D_∞ is a constant at "infinite time"(d^{-1}), D_1 is a constant "intercept" at 1-time unit (d^{-1}), and n_{PLE} is a time exponent.

This approach offers the advantage of accurately matching production data in both flow regimes, without being overly sensitive, as noted by [29]. It also methods a range of different flow periods typically seen in shale gas reservoirs, including linear, bilinear, BDF, and transient linear flow regimes, as discussed in [19, 32]. Figure 4 illustrates the typical production behavior of shale reservoirs, with a fast decline rate in the early stages, a long-tail of production with a smaller decline rate in the late stages.

However, it should be noted that the PLE model can sometimes provide overly optimistic recovery estimates, even when the flow regime changes within the first ten years of production, as observed by [36]. Additionally, the model assumes that BDF is reached at a late time when reservoir properties become constant, which can make it challenging to accurately determine ($D\infty$) and obtain (Di) with noisy production data, as noted in [12]. Moreover, the PLE model involves solving for four parameters, which can be cumbersome, although these extra variables do allow flexibility to match transient and BDF behaviors, as discussed in [14, 18, 31]

Duong approach

Tight or shale gas reservoirs are characterized by their extremely low permeability, which results in a prolonged production period that could last for years in transient flow. The fracture system contributes significantly to the production, while the matrix contribution is negligible. To model the decline behavior of shale gas, Duong proposed an approach that is not dependent on fracture type (finite conductivity, infinite conductivity, natural or artificial), well type (vertical or horizontal), or completion type (single or multi-stage) [9, 10]. This model was developed under the assumption of constant bottom-hole flowing pressure [24], and its mathematical expression is shown in Eqs. 18 and 19, as presented in the works of [14, 22].

$$\frac{q_g}{G_P} = at^{-m}, \tag{18}$$

$$q = q_i \cdot t^{-m_D} \exp\left[\frac{a_D}{1 - m_D}\left(t^{1-m_D} - 1\right)\right], \tag{19}$$

where; a_D is an intercept constant (d^{-1}), and m_D is the slope.

It is important to note that the exponent "m" is always a positive number in all calculations. However, Paryani et al. [31] highlighted two issues related to this model: extended shut-in periods and water breakthrough. To accurately determine the (a) and (m) parameters, proper initialization of the rate after shut-ins is required, and water breakthrough can cause a sudden decrease in the production rate, resulting in an increase in the permeants value.

Similar to the PLE model, changes in flow regimes within the first ten years can result in an overestimation of the EUR using this model. However, the model provides a reasonable recovery estimation for non-horizontal non-hydraulic fractured shale wells, as observed by Vanorsdale [36].

Extended Exponential Decline Curve Model (EEDCM)

Zhang et al. presented a revised model with easily adjustable parameters to represent the production behavior of shale gas wells during both transient and boundary-dominated flow periods [45]. The proposed model appears to be a modification of Fetkovich et al.'s work, as demonstrated in Eq.20 [11]. In their modification, Zhang et al. updated the (a) parameter by including the sum of two constants to represent the decline rate, which is shown in Eq.21. The proposed model's mathematical expression is presented in Eq.22.

$$q = q_i e^{-at}, \tag{20}$$

$$a = \beta_l + \beta_e, \tag{21}$$

$$q = q_i \cdot \exp\left[-\left(\beta_l + \beta_e \cdot e^{-tn_E}\right)t\right], \tag{22}$$

where β_e is a constant to account for the early (fully transient) period, β_l is a constant to account for the later period, and n_E is an empirical exponent, with a recommended range of 0–0.7.

It is worth noting that βe should be greater than βl, as recommended, since βe accounts for late-time behavior, whereas βl does not affect the fitting of the production data and is therefore fixed. The model fitting process results in a smooth curve that encompasses the entire flow system [23, 24, 31].

For short-term production data, this approach is the most effective for forecasting in the absence of recurring variations [27]. However, one assumption inherited from Fetkovich's model is the constant bottom hole pressure, which is not applicable in shale reservoirs.

Fractional Decline-Curve (FDC)

The long-tail production behavior, also known as the long-range dependence phenomenon, has been extensively studied in geology and is statistically referred to as anomalous diffusion [30]. To predict this anomalous diffusion, fractional diffusion equations are used to model this phenomenon, and this concept has been applied to various scientific fields [5, 6, 25, 26, 46]. Shale gas production decline curves exhibit the same long-tail behavior, and Zuo et al. utilized this concept to develop a new approach for matching shale gas well production [47]. The proposed model is presented in Eq.23, and its simplified form is shown in Eq. 24.

$$q = m \sum_{k=0}^{\infty} \frac{(-\lambda t^{\alpha})^k}{\Gamma(\alpha k + 1)}, \tag{23}$$

$$q = \frac{q_{\mathrm{i}}}{\lambda \cdot \Gamma(1-\alpha)t^{a}}, \tag{24}$$

where; λ is the eigenvalue, α is a fitting coefficient (dimensionless), and $\Gamma(t)$ is the gamma function.

The gamma function concept has been utilized in other decline curve methods, such as the Weng and Weibull methods. These methods are dependent on statistical behavior as they are considered probability density functions. They are sensitive to the size and range of the data sample used for fitting, making the available production data size critical before forecasting production using these methods.

Hyperbolic–Exponential Hybrid Decline (HEHD) model

Yadong et al. suggested a hybrid model that combines hyperbolic and exponential decline methods to address the early-time hyperbolic decline and late-time exponential decline observed in production data. The hybrid model, shown in Eq. 25 [44], was found to provide better fitting accuracy for historical data than the PLE model by 8.1%. This model is primarily intended for application to horizontal shale gas wells [12].

$$q = q_{\mathrm{i}} \cdot \frac{\exp(-D_{\infty} \cdot t)}{(1 + m_{\mathrm{HEHD}} \cdot D_{\mathrm{i}} \cdot \mathrm{t})^{\left(1-\frac{D_{\infty}}{D_{\mathrm{i}}}\right)/m_{\mathrm{HEHD}}}}, \tag{25}$$

where; m_{HEHD} is a time coefficient (dimensionless).

Wang model

Wang et al. developed a new model, which they believed was more general than Duong's model and could estimate estimated ultimate recovery (EUR) for short or long production periods. The model, expressed in Eq. 26 [37], is based on the same assumptions as Duong's model and takes into account the time effect on the fracture time exponent.

$$q = q_i \cdot \exp\left[-\lambda_W (\ln t)^2\right], \tag{26}$$

where; λ_{W} is an empirical coefficient (dimensionless).

Plotting Ln(q) versus (Ln t)^2 on a Cartesian plot would yield a straight line with the slope = —$\lambda_{\mathrm{W},}$ which is later on used to calculate the G_p and EUR. skin effect can also be inferred from this plot.

Variable Decline Modified Arps (VDMA) model

Gupta et al. proposed a modified Arps's model to account for various flow regimes in fractured wells during the early stages of production [12]. They suggested that the decline in fracture permeability with time or any linear flow could be modeled using the power-law exponential (PLE) approach. Additionally, they approximated the boundary dominated flow (BDF) regime to either a constant decline rate or an exponential model. The modified model introduced variable exponential decline in Arps's constant decline model, as shown in Eq. 27.

$$q = q_{\mathrm{i}} \cdot \exp\left[-D_{\mathrm{i}} \cdot t^{(1-n_{\mathrm{VDMA}})}\right], \tag{27}$$

where; n_{VDMA} is a decline-rate exponent

Uncertainties related to decline curve studies

There are numerous uncertainties associated with the application of DCA methods, particularly for shale gas wells, resulting in the creation of various empirical correlations to match the decline behavior. Yehia et al. [40–42] summarizes the uncertainties associated with DCA into the following sources:

The uncertainties related to determining the parameters

The parameters of each decline curve model play a crucial role in controlling the initial decline value, the curvature shape, and the declining behavior. However, determining these parameters can be a challenging task. Several authors have proposed graphical methods for this purpose, but these methods can be time-consuming, particularly when dealing with a large number of wells. Non-linear regression is the most commonly used technique to determine these parameters, but it is also a challenging task.

The size of the historical data

DCA is based on fitting the production history before predicting the EUR. The larger the data the more trust we have in the declining trend and the more reliable the prediction using a certain DCA model. However, some methods are much more sensitive to the data size than others. Table 1 categorized some of these methods.

The flow regime variations through the production life

The behavior of decline is affected by the type of flow regime present in a reservoir. In conventional reservoirs, BDF is the dominant flow, while unconventional reservoirs are dominated by transient flow, which can last for a significant period before transitioning to BDF. Unconventional reservoirs often exhibit bilinear and linear flow regimes. The transition from one transient flow regime to another is associated with a significant change in decline behavior. Not all methods can consider the various flow regimes, leading to differences and modifications in their structure. Proposed methods can be classified based on flow regime characteristics as shown in Table 2. These methods are often correlated and can be transformed into each other under specific conditions [24].

Table 1 Decline curve methods' sensitivities to data size and data quality

Sensitivity to Data Size		
Less Sensitive	**Moderate Sensitive**	**Highly Sensitive**
K-Model	Arps	Weng
Duong	MLM	Weibull
HEHD	T-Model	Hsieh
Wang	LGM	SEPD
	FDC	EEDCA
		VDMA

Table 2 Classifying the empirical DCA methods based on considering the flow behavior or not

Model consider the flow behavior	Model do not consider the flow behavior
Hsieh (2001)	Arps (1945
PLE (2008)	MLM (1956)
Duong (2010, 2011)	K (1970)
EEDCA (2015)	Weng (1984)
FDC (2016)	Weibull (1995)
HEHD (2016)	SEPD (2010)
Wang (2017)	LGM (2011)
VDMA (2018)	

Cases study

This study is based on actual data that was released in 2021 on the SPE official website. The data consists of more than forty wells of dry gas in shale gas reservoirs. Three wells were selected. The selection was based on choosing wells from different field, different reservoirs and different flow regime time. The reservoirs are Haynesville Shale (Lorikeet Field), Marcellus Shale (Ostrich Field), and Marcellus-Upper Shale (Penguin Field).

Calculation and matching procedure

This is a description of the calculation and matching procedure for each model,

1. A log–log plot of rate versus time is done for each well to know their flow behavior and the time at which the flow reached the boundary.
2. All methods were programmed using the visual basic in excel.
3. The matching procedure for each case was performed using a semi-log basis (log q vs. t).
4. In each of the data cases, data size was tested using different number of days for fitting data (30,90,180,360,500,800, and 1000 days).
5. To determine the best fit for each case, the least squares method was employed, which involved minimizing the sum of the squared differences between the actual values and the calculated values of the field data.
6. Comparison and analysis is done for all the three cases in order to investigate the effect of the data size, the effect of the time at which the flow reaches the boundary (early during the first year or late).

Results and discussion

Case 1_Bilinear-linear-boundary dominated flow

Well_1 is a dry gas well that is located in US and produces from Haynesville shale with an initial rate of 11,900 MSCFD. A log–log plot of rate versus time indicates the presence of all the difference flow regimes together as shown in Fig. 2. A bilinear quarter slope is observed during the early time followed by linear flow regimes of -1/2 slope. Also, it's shown that the fluid reaches the boundary at early time during the first year of production.

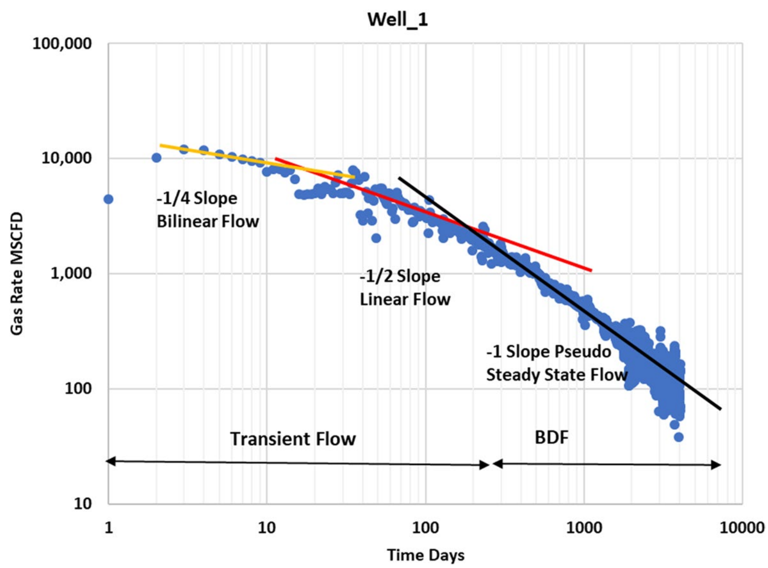

Fig. 2 Identifying the different flow regimes of well_1 based on the slope value on the log–log plots

A semilog plot of rate versus time was plotted for different data size (30,90,180,360,500,800,1000 days) to test the response of the different methods as shown in Fig. 3A to G. The arrows on the figures indicates the method that gave the best fitting.

The methods that consider the flow behavior and the methods that neglect the flow behavior are analyzed based on the data length, reserves estimation and the time at which the flow reaches the boundary as shown in Tables 3 and 4.

Case 2_ Linear-BDF

Well_2 is a dry gas well that is located in US and produces from Marcellus shale with an initial rate of 6,800 MSCFD. A log–log plot of rate versus time indicates the presence of linear flow followed by a BDF as shown in Fig. 4. A linear flow regime of -1/2 slope is observed during the first three years of production. Also, it's shown that the fluid reaches the boundary at the end of the third year of production.

A semilog plot of rate versus time was plotted for different data size (30,90,360,500,800,1000 days) to test the response of the different methods as shown in Fig. 5A to F. The arrows on the figures indicates the method that gave the best fitting.

The methods that consider the flow behavior and the methods that neglect the flow behavior are analyzed based on the data length, reserves estimation and the time at which the flow reaches the boundary as shown in Tables 5 and 6.

Case 3_ bilinear-linear flow

Well_3 is a dry gas well that is located in US and produces from Marcellus-Upper shale with an initial rate of 9,200 MSCFD. A log–log plot of rate versus time indicates that the well still in the transient flow and didn't reach the boundary as shown in Fig. 6. A bilinear flow with quarter slope is observed during the early time followed by linear flow regimes of -1/2 slope.

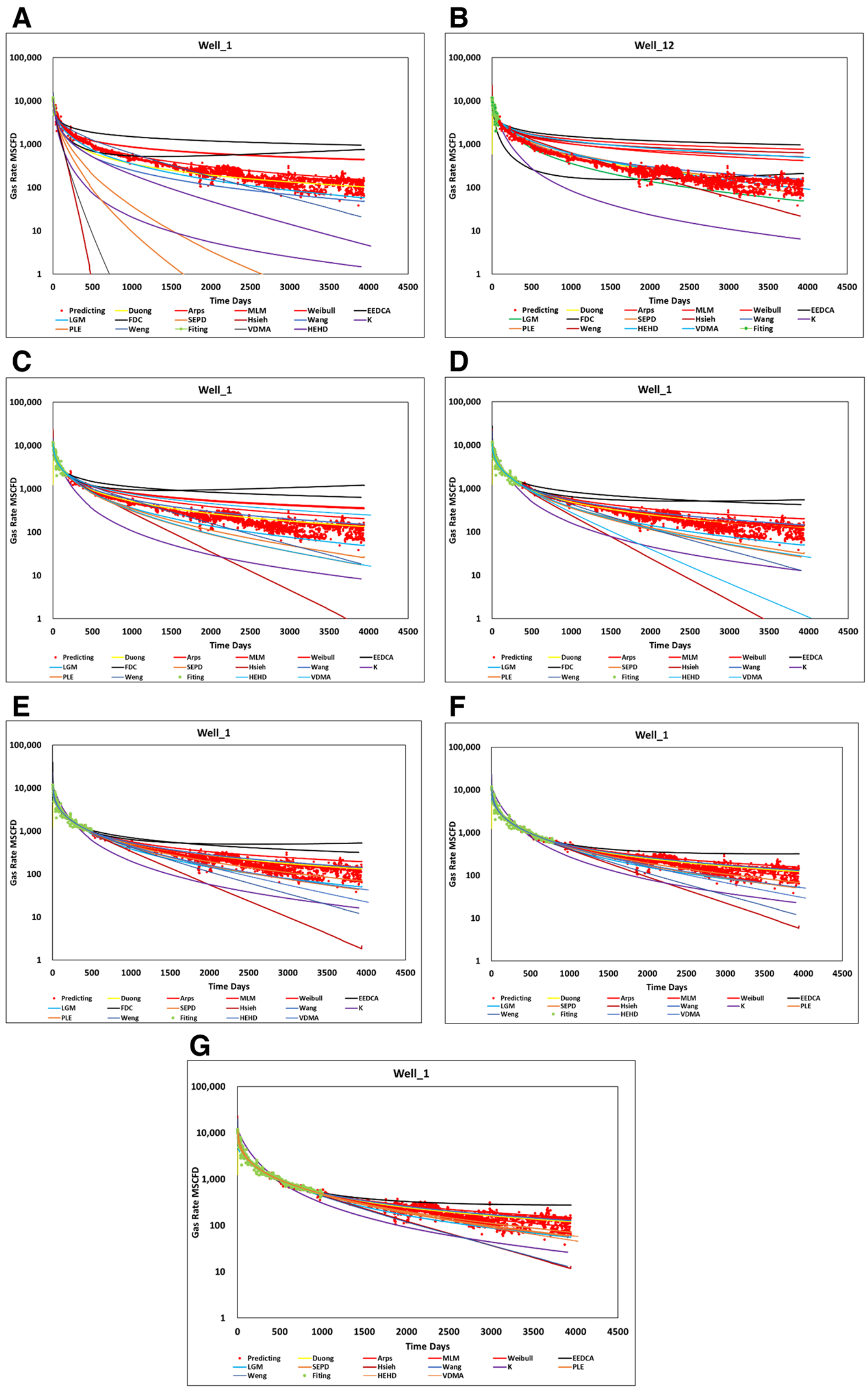

Fig. 3 **a** Fitting 30 Days (Transient Flow) of Data length. **b** Fitting 90 Days (Transient Flow) of Data length. **C** Fitting 180 Days (Transient Flow) of Data length. **D** Fitting 360 Days (BDF) of Data length. **E** Fitting 500 Days (BDF) of Data length. **F** Fitting 1000 Days (BDF) of Data length. **G** Fitting 1000 Days (BDF) of Data length

Table 3 Decline curve models of well_1 that do not consider the flow behavior of shale gas for prediction performance according to data size

Date length/d	30	90	180	360	500	800	1000
Arps	+	+	+	+	+	+	+
MLM	+	+	+	+	+	+	+
Weng	-	-	-	-	-	-	-
K	-	-	-	-	-	-	-
SEPD	-	+	-	-	*	*	*
Weibull	+	+	+	+	*	*	*
LGM	-	-	-	-	-	-	-

(a) (+) means that the model tends to be over estimator of the reserve. (-) means that the model tends to be under estimator of the reserve. (*) means that the model tends to be good estimator of the reserve

(b) Yellow is recommended methods, blue is not recommended methods, green is transient flow period, pink is BDF

Table 4 Decline curve models of well_1 that consider the flow behavior of shale gas for prediction performance according to data size

Date length/d	30	90	180	360	500	800	1000
HSIEH	-	-	-	-	-	-	-
PLE	-	*	-	-	-	-	-
Duong	*	*	*	*	*	*	*
EEDCA	-	+	+	+	+	+	+
FDC	-	+	+	+	+	+	+
HEHD	-	+	+	-	-	-	-
VDMA	-	*	-	-	-	-	-
Wang	-	+	+	+	*	*	*

(a) (+) means that the model tends to be over estimator of the reserve. (-) means that the model tends to be under estimator of the reserve. (*) means that the model tends to be good estimator of the reserve

(b) Yellow is recommended methods, blue is not recommended methods, green is transient flow period, pink is BDF

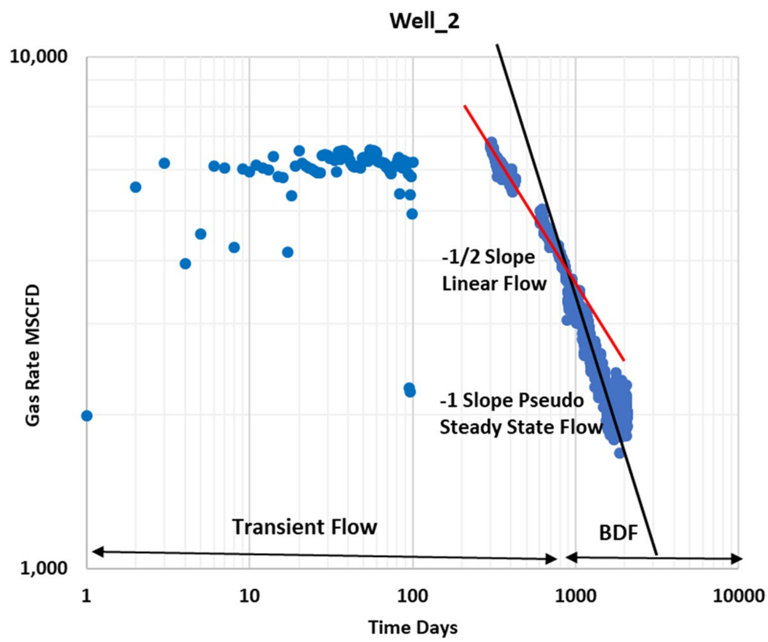

Fig. 4 Identifying the different flow regimes of well_2 based on the slope value on the log–log plots

A semilog plot of rate versus time was plotted for different data size (30,90,360,500 days) to test the response of the different methods as shown in Fig. 7A to D. The arrows on the figures indicates the method that gave the best fitting.

The methods that consider the flow behavior and the methods that neglect the flow behavior are analyzed based on the data length, reserves estimation and the time at which the flow reaches the boundary as shown in Tables 7 and 8.

Conclusions

This paper provides a review of 14 decline curve analysis (DCA) models that have been developed for both conventional and unconventional hydrocarbon wells. Each model has its own unique structure and set of assumptions. Based on this review and on the cases study, the following conclusions can be drawn:

(1) All methods were developed under specific assumptions or conditions; it is not possible to utilize a single or general model for all types of reservoirs or decline modes. For instance, Duong's model was developed with the assumption of constant bottom-hole pressure, and LGM was specifically developed for extremely tight shale gas unconventional reservoirs.

(2) Certain models can be transformed into others under specific assumptions. For instance, the MLM model can be transformed into Arps's hyperbolic model, while the SEPD model can be transformed into the PLE model.

(3) Although many models account for different flow regimes, particularly for unconventional reservoirs, this does not necessarily ensure reliable predictions, even when the historical data fit well.

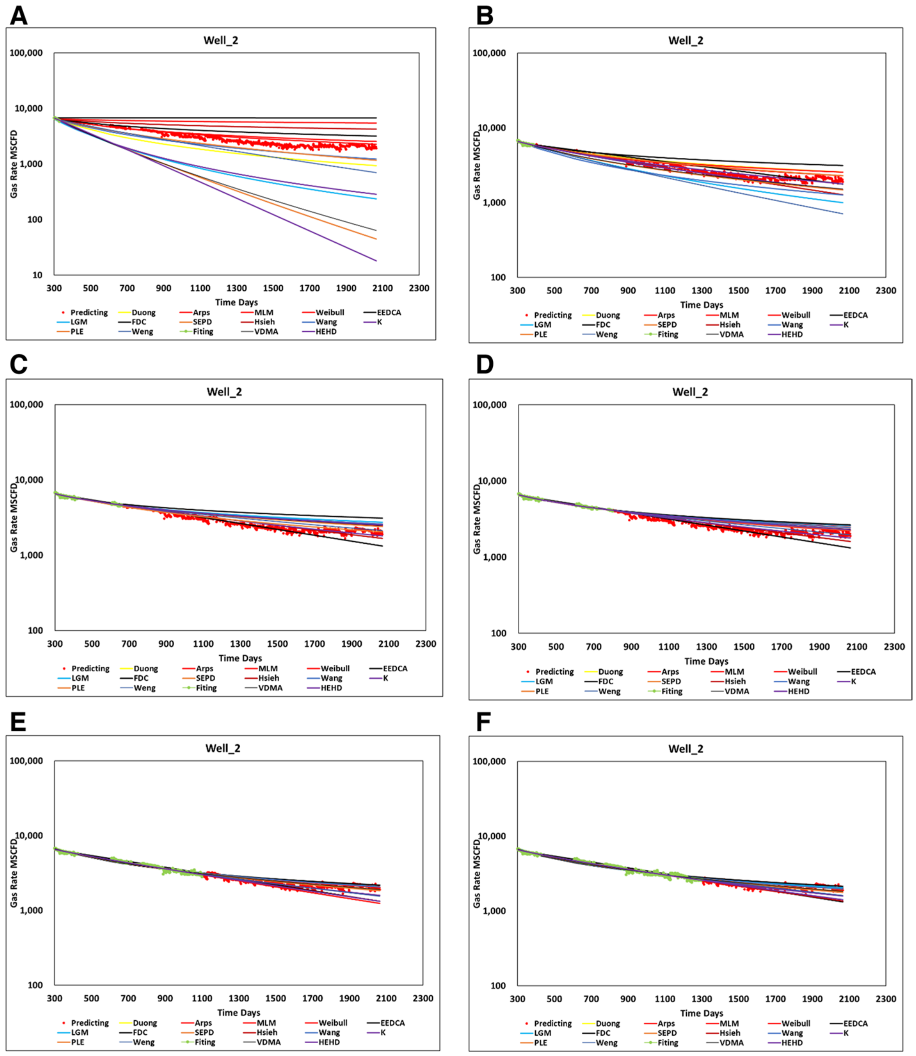

Fig. 5 **A** Fitting 30 Days (Transient Flow) of Data length. **B** Fitting 90 Days (Transient Flow) of Data length. **C** Fitting 360 Days (Transient Flow) of Data length. **D** Fitting 500 Days (Transient Flow) of Data length. **E** Fitting 800 Days (Transient Flow) of Data length. **F** Fitting 1000 Days (BDF) of Data length

Table 5 Decline curve models of well_2 that do not consider the flow behavior of shale gas for prediction performance according to data size

Date length/d	30	90	360	500	800	1000
Arps	-	+	+	+	+	*
MLM	-	+	+	+	-	-
Weng	-	-	*	*	-	-
K	-	-	-	-	-	-
SEPD	-	+	+	+	*	-
Weibull	-	+	+	+	*	*
LGM	-	-	+	+	+	*

(a) (+) means that the model tends to be over estimator of the reserve. (-) means that the model tends to be under estimator of the reserve. (*) means that the model tends to be good estimator of the reserve

(b) Yellow is recommended methods, blue is not recommended methods, green is transient flow period, pink is BDF

Table 6 Decline curve models of well_2 that consider the flow behavior of shale gas for prediction performance according to data size

Date length/d	30	90	360	500	800	1000
HSIEH	-	-	-	-	-	-
PLE	-	-	*	*	*	-
Duong	-	+	+	+	*	*
EEDCA	-	-	-	-	-	-
FDC	-	+	+	+	+	*
HEHD	-	*	+	+	-	-
VDMA	-	-	+	+	-	-
Wang	-	-	+	+	*	*

(a) (+) means that the model tends to be over estimator of the reserve. (-) means that the model tends to be under estimator of the reserve. (*) means that the model tends to be good estimator of the reserve

(b) Yellow is recommended methods, blue is not recommended methods, green is transient flow period, pink is BDF

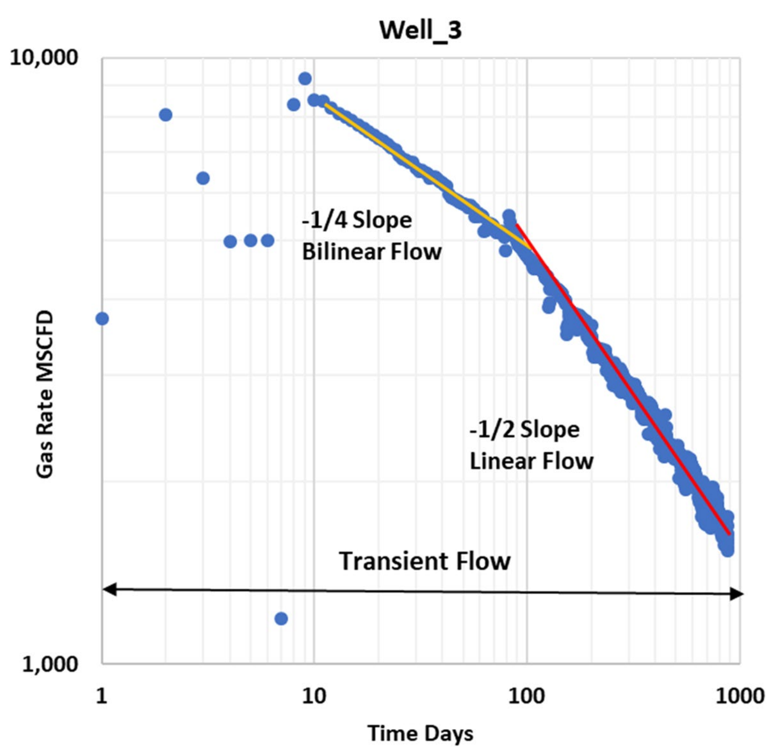

Fig. 6 Identifying the different flow regimes of well_3 based on the slope value on the log–log plots

(4) For the case study of the Haynesville shale, when the BDF is reaches at early time (at the first year), some methods like Duong and Wang gives a good estimation of the reserves from the early time of matching. However, SEPD and Weibull methods gives a good estimation of the reserve after 360 days.

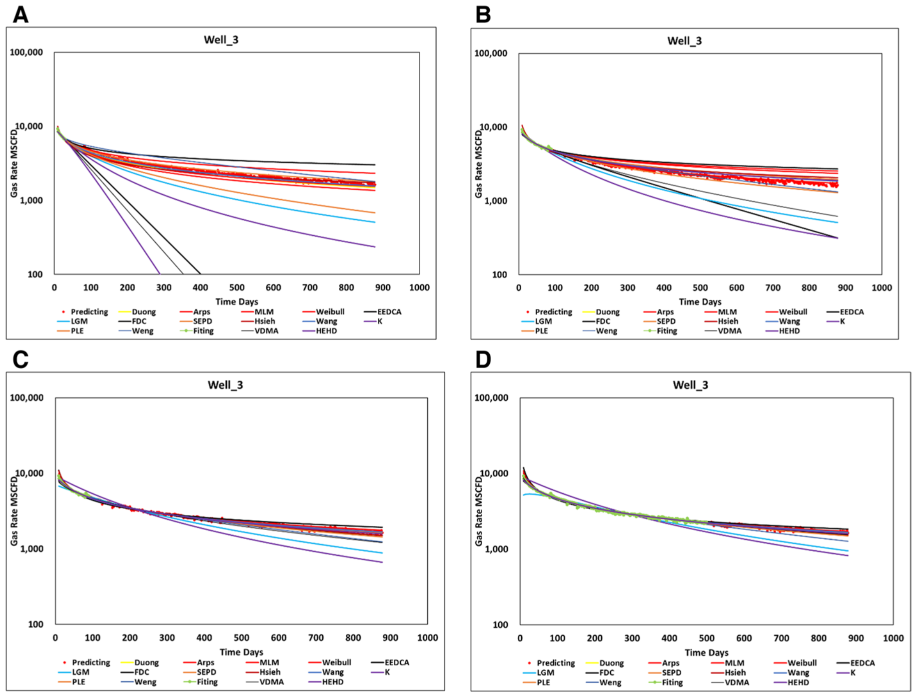

Fig. 7 **A** Fitting 30 Days (Transient Flow) of Data length. **B** Fitting 90 Days (Transient Flow) of Data length. **C** Fitting 360 Days (Transient Flow) of Data length. **D** Fitting 500 Days (Transient Flow) of Data length

Table 7 Decline curve models of well_3 that do not consider the flow behavior of shale gas for prediction performance according to data size

Date length/d	30	90	360	500
Arps	-	*	*	*
MLM	-	+	*	*
Weng	-	-	-	-
K	-	-	-	-
SEPD	+	+	*	*
Weibull	+	+	+	*
LGM	-	-	-	-

(a) (+) means that the model tends to be over estimator of the reserve. (-) means that the model tends to be under estimator of the reserve. (*) means that the model tends to be good estimator of the reserve

(b) Yellow is recommended methods, blue is not recommended methods, green is transient flow period, pink is BDF

(5) For the case study of the Marcellus shale, when the BDF is reaches at later time (at the end of the third year), most of the methods started to give a good matching for the data after 360 days. PLE is the most recommended method for this case. Also, Duong and Wang gives a good matching but at late time when the flow started to reach the boundary.

(6) For the case study of Marcellus-Upper when the flow still in the transient regime. Duong, Wang and SEPD methods gives a good matching after 360 days which can indicate that the flow is closer to the boundary.

Table 8 Decline curve models of well_3 that consider the flow behavior of shale gas for prediction performance according to data size

Date length/d	30	90	360	500
HSIEH	+	+	-	-
PLE	+	-	-	-
Duong	-	+	*	*
EEDCA	-	-		*
FDC	+	+	+	+
HEHD	-	+	*	*
VDMA	-	-	-	-
Wang	-	+	*	*

(a) (+) means that the model tends to be over estimator of the reserve. (-) means that the model tends to be under estimator of the reserve. (*) means that the model tends to be good estimator of the reserve

(b) Yellow is recommended methods, blue is not recommended methods, green is transient flow period, pink is BDF

(7) From all the three cases it can be concluded that as the boundary is reached at the early time as more methods will give a good match and a good reserve estimation. Also, as the boundary is reached at late time, in order to have a good reserve estimation, 360 days of data length is needed at least in order to fit the data.

Recommendations

There are several uncertainties associated with DCA, including the fitting process, data size, and data quality. To ensure effective DCA, the following recommendations are suggested:

(1) It is highly recommended to use multiple DCA models and compare their forecasting reliability. Probabilistic DCA is also recommended to create intervals of confidence rather than a deterministic value of the reserve.
(2) The Ordinary least square (OLS) fitting technique should be used with pessimistic DCA models that underestimate the reserve, as it results in optimistic forecasting. Conversely, the weighted least square regression (WLS) fitting technique should be used with optimistic DCA models that overestimate the reserve, as it results in pessimistic forecasting.
(3) Improving data quality before conducting DCA by removing noise can enhance the model's goodness of fitting and prediction reliability.

Nomenclature

b Decline-Curve Exponent
D Decline Rate (Day^{-1})
D_i Initial Decline Rate (Day^{-1})
Gp Gas Cumulative Production (Mscf)
q Gas Flow Rate (Mscf/D)
t Time (day)

Abbreviations

BDF	Boundary Dominated Flow
BHP	Bottom Hole Pressure
DCA	Decline Curve Analysis
EEDCA	Extended Exponential Decline Curve
EUR	Estimated Ultimate Recovery
FDC	Fractional Decline-Curve
HEHD	Hyperbolic –Exponential Hybrid Decline
LGM	Logistic Growth Model
MLM	Matthews-Lefkovits Model
OLS	Ordinary least square
PLE	Power-law Equation
SEPD	Stretched Exponential Decline Model
VDMA	Variable Decline Modified Arps
WLS	Weighted least square

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Authors' contributions
S.Coutry designed the research project, conducted the data collection and analysis, and drafted the manuscript. M.Tantawy and S.Fadel provided critical feedback and revisions to the manuscript. All authors reviewed and approved the final version of the manuscript for submission.

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Availability of data and materials
The data used in the three case studies are online data that can be accessed through this link https://www.spe.org/datasets.

Declarations

Competing interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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