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# Effect of zones' dimensions and geometry on seepage through zoned earth dams

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## Abstract

The seepage study through earth dams is very essential for the design and construction processes of such dams to ensure the needed safety and efficient performance. The present study focuses on the seepage flow through zoned embankment dams by introducing a numerical analysis using the Seep/w numerical model. The main objective of the study is to investigate the different effects of the dam zones' thickness and side slopes on seepage through such dams to achieve the most suitable dimensions and geometry of the different zones. First, the Seep/w is used to analyze the problem of seepage through earth dams with an internal core. The present obtained results and the results of other previous experimental and analytical studies are almost close to each other. The present work proves that the best relative thickness of the inner, transition, and outer zones ( $t_1:t_2:t_3$ ) according to the minimum seepage and cost of the used materials is 2:1.5:1.5 respectively. At the same time, it is proven that the reasonable optimum side slopes (H:V) of the inner, transition, and outer zones are 1:1.75, 1.25:1, and 3.75:1 respectively.

**Keywords:** Earth Dam, Seepage, Zones' Geometry, Seep/W

## Introduction

Earth dams are constructed across rivers and irrigation canals for different technical purposes. Seepage flow is one of the most essential concerns for the design process of such dams. It was concluded that more than 30% of dams' failures were due to uncontrolled seepage [1, 2]. Many studies have investigated the seepage flow problem through earth dams and tried to control that seepage and avoid such failure. Zhang et al. [3] investigated the causes of failures for more than 1600 dams all over the world, including details of the dams, and their failures. They focused on failures of earth dams, which were about 66% of the cases. The most common causes of earth dam failures were piping and overtopping for homogeneous and zoned embankment dams. Flores and Lopez [4] studied erosion due to seepage in earth dams. Many recommendations were presented for preventing the failure such as obtaining the best selection of available materials in addition to using transition zones between the coarse and fine materials.

Boroomand and Mohammadi [5] analyzed the seepage of Alborz dam in Iran. It was found that the seepage quantity would increase significantly if the dam was without a core, and the core was very necessary to decrease the seepage discharge through the

dam. Sazzad and Islam [6] made a comprehensive study of the methods of seepage control for earth dams. It was noticed that the trapezoidal shape of the internal core was better than other shapes. Galindo et al. [7] carried out a study of different materials for the dam's core, using a wide range of permeability variations. Design charts were presented to determine the failure probability of the dam according to the different used materials. Sazzad and Alam [8] studied the seepage characteristics of different types of earth dams. They found that the pore water pressure decreased linearly with the dam distance for the homogeneous earth dam, while the pore water pressure did not follow any regular trend in the case of the diaphragm or zoned earth dams.

Fattah et al. [9] studied the seepage of a zoned earth dam using a finite element method. A case study was considered to be Al-Adhaim dam in Iraq. It was concluded that the clay core had an important effect on decreasing the exit gradient, which may increase significantly if the core did not exist. The sloping core of Al-Adhaim dam was the best design for the core than other choices since it permitted the minimum values of seepage discharge and provided the lowest gradients. Sazzad et al. [10] made a comparison between numerical and analytical analysis of seepage flow through earth dams. The seepage discharge obtained from the numerical analysis was close to that from the analytical analysis by Fakhari and Ghanbari [11]. A remarkable decrease in the seepage discharge was noticed with a core in the earth dam. The upstream and downstream slopes had a slight effect on the seepage discharge by using an internal core.

Sazzad et al. [12] investigated seepage through earth dams numerically and analytically for various parameters. Upstream and downstream slopes did not affect the seepage by using a clay core. Hassan [13] used a genetic algorithm technique and finite element modeling to obtain the optimum inclination angle and location of a cut-off wall under hydraulic structures. Khassaf and Madhlloom [14] studied the effect of an impervious core on seepage through a zoned earth dam (Khassa Chai dam) in Iraq. It was concluded that the core was very necessary for the dam to decrease the phreatic surface, pressure head, and seepage discharge through the dam body, and the sloping core of Khassa Chai dam was the best for the core than other slopes as it causes the minimum seepage discharge values. Mazaheri et al. [15] investigated the effect of the horizontal drain dimensions through Marvak earth dam in Iran using neural networks. The obtained results indicated that the internal friction angle of the drain's material and the side slope of the earth dam had the main effect on determining the dam's factor of safety. Salem et al. [16] analyzed the seepage flow through embankment dams with an internal core. They concluded that decreasing the core permeability gave a reduction in the seepage discharge, gradient, and pore water pressure. Increasing the core width caused a decrease in the seepage quantity. Hassan [17] investigated the effect of seepage through anisotropic porous soil under hydraulic structures using different water heads and anisotropy degrees. Komasi et al. [18] studied the main factors affecting the factor of safety of a homogeneous earth dam with the use of Geostudio numerical models and artificial neural network. Mostafa and Shen [19] investigated the seepage flow through zoned earth dams. Decreasing the inner zone hydraulic conductivity caused a significant decrease in the different seepage parameters.

Abdel-Kawy et al. [20] investigated the effects of core properties on seepage behavior through earth dams. The wedge-shaped core was the best shape to reduce the seepage

discharge and the seepage line. The vertical core came next, followed by the upstream and downstream inclined cores, which had almost the same effect. The core upper width ratio was effective for all core shapes except the wedge-shaped one. Alzamily and Abed [21] carried out experimental and theoretical studies of seepage flow through earth dams using a special core. It was found that the sandy soil model with some additives had a close permeability to the permeability of the clay model. Koś et al. [22] investigated using sediments of dam reservoirs for constructing earthworks. It was concluded that by using a core made of sediments, the seepage volume on the downstream area was significantly decreased than that with the case of no core. Beiranvand and Rajaei [23] carried out a comprehensive study on intelligent models for modeling the seepage flow in earth dams and found that the applications of these models were efficient in modeling the seepage and the pore water pressure. Hassan et al. [24] found that the optimum location ratios of the upstream and downstream cut-offs were about 0.22 and 0.94 from the floor’s toe of the hydraulic structure respectively.

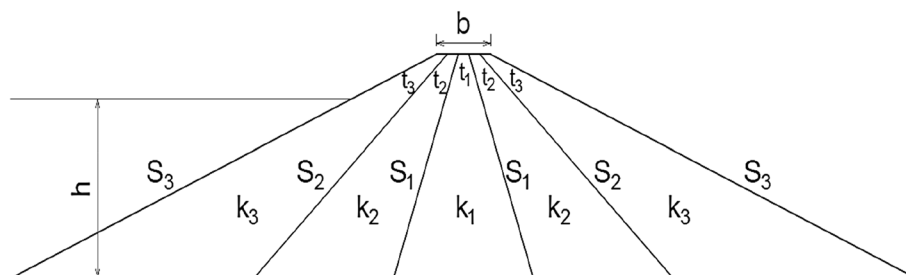
In the present study, the Seep/w numerical model is used to investigate the seepage through a zoned earth dam founded on impervious foundations. The numerical models include the effect of various dimensions and geometry of the different zones. The study aims to achieve the most suitable relative thickness and side slopes of the different zones. Selecting the optimum dimensions and geometry of the different zones according to the minimum seepage and cost of the used material has not been widely studied as noticed from the literature review.

**Dimensional analysis**

The general relationship for the variables involved in the research as shown in Fig. 1 can be written as:

$$f(\rho, g, b, h, q, k_1, k_2, k_3, y_1, y_2, t_1, t_2, t_3, S_1, S_2, S_3) = 0 \tag{1}$$

By using Buckingham’s  $\pi$  theorem and taking the gravitational acceleration and water density as constants, the relationship of the non-dimensional parameters can be expressed as:



**Fig. 1** Variables used for studying zoned earth dams. Where: b: top width of the dam. h: upstream water head acting on the dam.  $k_1$ : hydraulic conductivity of the inner zone.  $k_2$ : hydraulic conductivity of the transition zones.  $k_3$ : hydraulic conductivity of the outer zones. q: seepage discharge rate per unit length.  $S_1$ : side slopes of the inner zone.  $S_2$ : side slopes of the transition zones.  $S_3$ : side slopes of the outer zones.  $t_1$ : top thickness of the inner zone.  $t_2$ : top thickness of the transition zone.  $t_3$ : top thickness of the outer zone.  $y_1$ : phreatic level at the upstream side of the inner zone, taking the dam base as the datum.  $y_2$ : phreatic level at the downstream side of the inner zone, taking the dam base as the datum

$$f\left(\frac{b}{h}, \frac{q}{k_3 h}, \frac{k_1}{k_3}, \frac{k_2}{k_3}, \frac{y_1}{h}, \frac{y_2}{h}, \frac{t_1}{h}, \frac{t_2}{h}, \frac{t_3}{h}, S_1, S_2, S_3\right) = 0 \quad (2)$$

By assuming the upstream water head (h), top width of the dam (b), and different hydraulic conductivities to be constant, the equation between the different affecting parameters can be written as follows:

$$f\left(\frac{q}{k_3 h}, \frac{y_1}{h}, \frac{y_2}{h}, \frac{t_1}{b}, \frac{t_2}{b}, \frac{t_3}{b}, S_1, S_2, S_3\right) = 0 \quad (3)$$

## Methods

### Numerical modeling description

The main aim of this paper is to achieve the most suitable dimensions and geometry of the different zones according to the minimum seepage parameters and the cost of the used materials. In the present study, the numerical models are carried out using the Seep/w numerical model (a sub-program of Geo-Studio) [25]. The Seep/w is a finite element model which can simulate water flow through porous media. It is based on the water flow through saturated and unsaturated soils following Darcy's Law. Seep/w can be used to analyze groundwater movement in both steady and transient states. Seep/w was selected to be used in the study because of its reliable outputs according to the results of many previous seepage research. Also, Seep/w was verified by different previous research including experimental and field data such as Salem et al. [16] and Kachare and Jagtap [26] respectively, and it was suitable and gave close results for modeling seepage through earth dams. The general governing differential equation for two-dimensional seepage can be illustrated as:

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial h}{\partial y} \right) + Q = \frac{\partial \theta}{\partial t} \quad (4)$$

where: h is the total head,  $k_x$  and  $k_y$  are the hydraulic conductivities in the X and Y directions respectively, Q is the applied boundary flux,  $\theta$  is the volumetric water content, and t is the time.

The first step is to define the units and scale and sketch the cross-section. The regions of soils are defined with different material properties, then the boundary conditions can then be assigned. The sample functions method is used for the volumetric water content data point function. The coefficient of permeability is kept constant for saturated zones, but for unsaturated soil, it takes different values depending on the matric suction. The total head boundary condition is assigned to the upstream side ( $h = 20$  m), and the potential seepage boundary condition is applied to the downstream side, while the zero-pressure boundary condition is selected for the downstream toe. The finite element mesh pattern used in the numerical models is quads and triangles. The number of nodes and elements of the control model are 2157 and 2024 respectively. Seep/w outputs include plotting the seepage line, flow lines, and equipotential lines of the total head, displaying different hydraulic parameters such as pore water pressure, water velocity, and hydraulic gradient, and calculating the total seepage discharge.

### Zoned earth dam's dimensions

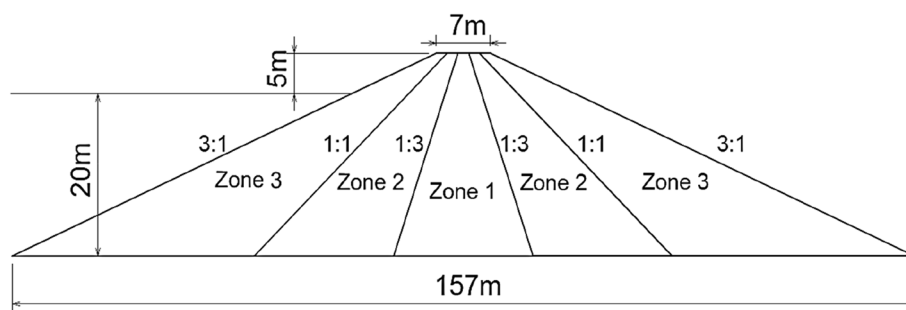
In the present study, a zoned earth dam is considered for the control model with 25 m height, 7 m top width, 20 m upstream water head (h) with 5 m freeboard and dry downstream side. The different dimensions of the dam's zones of the control model are selected according to the U.S. Bureau of Reclamation recommendations [27] and Fig. 2 shows a definition sketch for the introduced control model. The upstream and downstream slopes of the zoned earth dam are stable according to the literature recommendations. The soil foundation is assumed to be impermeable to avoid any seepage under the dam's body. The relative hydraulic conductivity between the inner, transition and outer zones ( $k_1:k_2:k_3$ ) is 0.0001:0.01:1 respectively. The thickness of each zone is constant. The different side slopes (H:V) are 1:3, 1:1, and 3:1 for the inner, transition, and outer zones respectively.

### Numerical modeling program

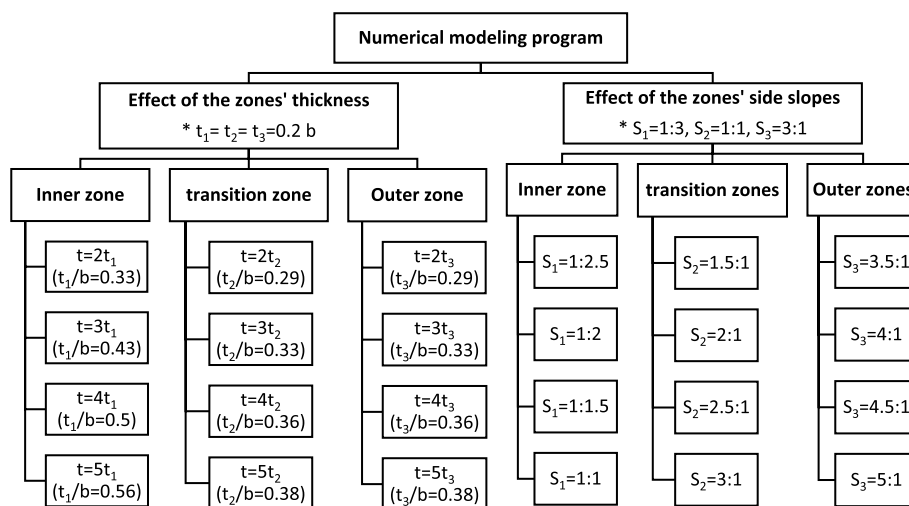
At the beginning of the numerical work, the control model is studied. Then, the study includes two main stages, the first is studying the effect of the different zones' thickness, while the second is studying the effects of the different zones' side slopes. The effect of the thickness of each zone is studied by increasing each thickness from 1 to 5 times the control one, and assuming the other thicknesses are constant. So, the thickness of the inner zone ranges from 1/5 to 5/9 of the crest width, while the thicknesses of the transition and outer zones range from 1/5 to 5/13 of the crest width. The effect of the side slopes of each zone is studied by increasing each side slope according to the ranges of the literature recommendations, and assuming the other side slopes are constant. Figure 3 shows the different numerical models of the present study.

### Verification of Seep/w results

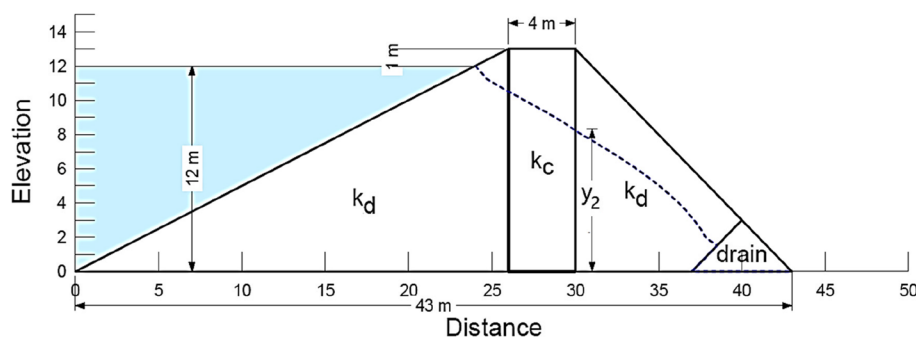
In this part, the Seep/w numerical model is used to analyze the problem of seepage through earth dams with an internal core based on an impervious base. Many studies considered different materials for the core and evaluate the effect of the impervious core properties on the seepage line and discharge. Rozanov [28] and [29] studied the earth dams with an internal core and introduced some equations to calculate the seepage discharge. Rezk [30] investigated experimentally the effect of the core characteristics on the seepage through earth dams. Rezk and Senoon [31] developed an analytical solution of the seepage problem through earth dams with an internal



**Fig. 2** Definition sketch for the introduced control model of zoned earth dams



**Fig. 3** Scheme of the different numerical models. \* Control model. Note: ratios of  $t_2$  and  $t_3$  are calculated for one zone (upstream or downstream)



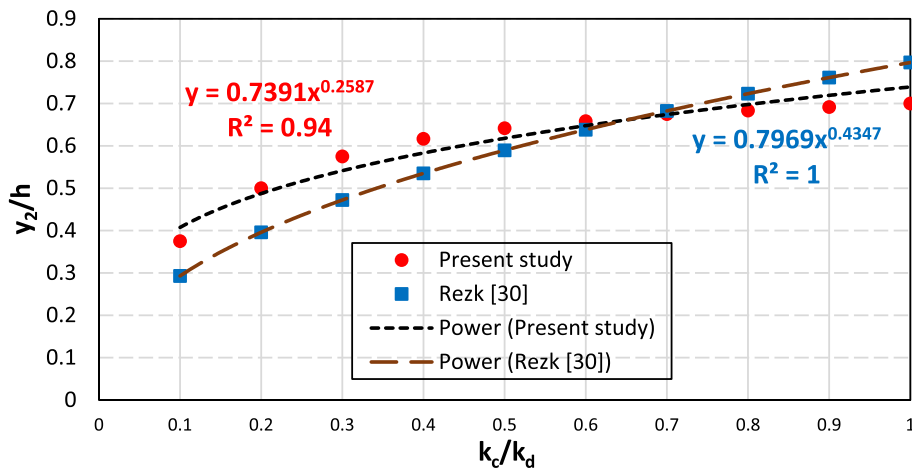
**Fig. 4** Definition sketch for an earth dam with an internal core ( $k_c/k_d = 0.8$ ) [31]

core as shown in Fig. 4. Design charts were presented in a dimensionless form to plot the phreatic line and determine the seepage discharge. A comparison is performed between the Seep/w results and the experimental results by Rezk [30]. The relation between the relative permeability coefficient ( $k_c/k_d$ ) and the phreatic line level downstream the core ( $y_2/h$ ) is plotted in Fig. 5. It is obvious that the phreatic lines in the two cases are almost close to each other

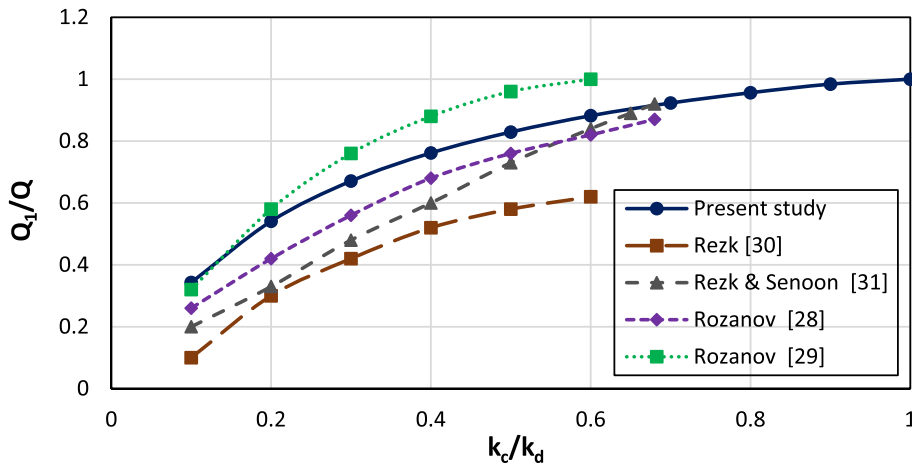
Figure 6 illustrates the relation between the relative permeability coefficient ( $k_c/k_d$ ) and the relative seepage discharge ( $Q_1/Q$ ) for different previous studies. It is clear that there is a close agreement between the present study and the other equations by Rozanov [28] and [29], and experimental and analytical results by Rezk [30], Rezk, and Senoon [31] respectively.

### Results and discussion

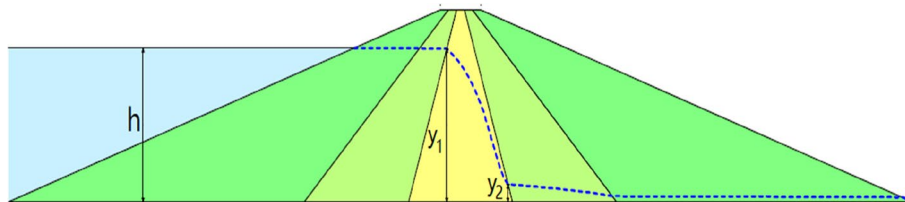
As it is mentioned before that the main objective of the present study is to investigate the different effects of the zones' dimensions and geometry through zoned earth dams. The study includes two main stages, the first is studying the effect of the different zones'



**Fig. 5** Relation between relative permeability coefficient ( $k_c/k_d$ ) and phreatic line level downstream core ( $y_2/h$ )



**Fig. 6** Relation between the relative permeability coefficient ( $k_c/k_d$ ) and the relative seepage discharge ( $Q_1/Q$ )



**Fig. 7** Phreatic line through the zoned earth dam

thickness, while the second is studying the effects of the different zones' side slopes to achieve the most suitable dimensions and geometry of such dams. Figure 7 shows the phreatic line levels through the zoned earth dam.

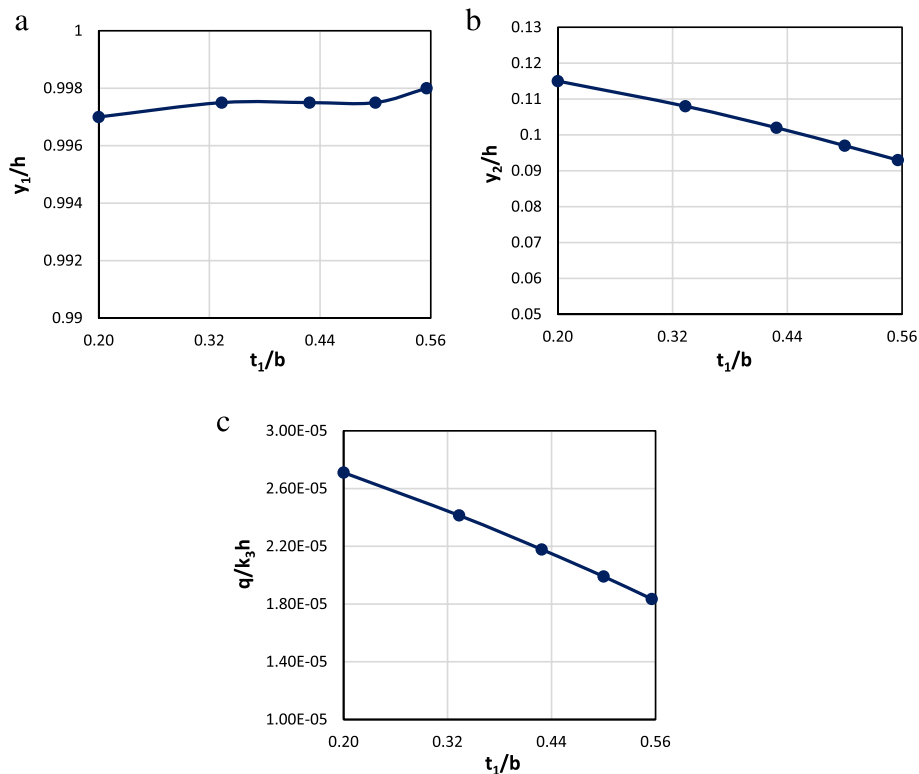
**Effect of the thickness of the zones on the seepage through zoned earth dams**

***Effect of the thickness of the inner zone (core)***

To determine the effect of the thickness of the inner zone on the seepage through the zoned earth dam, five models are tested by increasing the top thickness ( $t_1$ ) from 1 to 5 times the control one.

Figure 8-a shows the relation between the relative thickness of the inner zone ( $t_1/b$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). The figure illustrates that increasing the thickness of the inner zone causes a very slight increase in the phreatic line level at the upstream side of the inner zone, as increasing the thickness of the inner zone causes more flow obstruction which rises the phreatic line at the upstream side. The relation between the relative thickness of the inner zone ( $t_1/b$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ) is illustrated in Fig. 8-b, from which, increasing the thickness of the inner zone decreases the phreatic line level at the downstream side of the inner zone (By about 19%). This is due to the long distance taken by the water to reach the downstream side of the inner zone which causes more head loss.

The relation between the relative thickness of the inner zone ( $t_1/b$ ) and the seepage discharge ratio ( $q/k_3h$ ) is shown in Fig. 8-c. It is obvious that the seepage discharge decreases significantly with increasing the thickness of the inner zone (By 48%).



**Fig. 8** **a** Relation between the relative thickness of the inner zone ( $t_1/b$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). **b** Relation between the relative thickness of the inner zone ( $t_1/b$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ). **c** Relation between the relative thickness of the inner zone ( $t_1/b$ ) and the discharge ratio ( $q/k_3h$ )

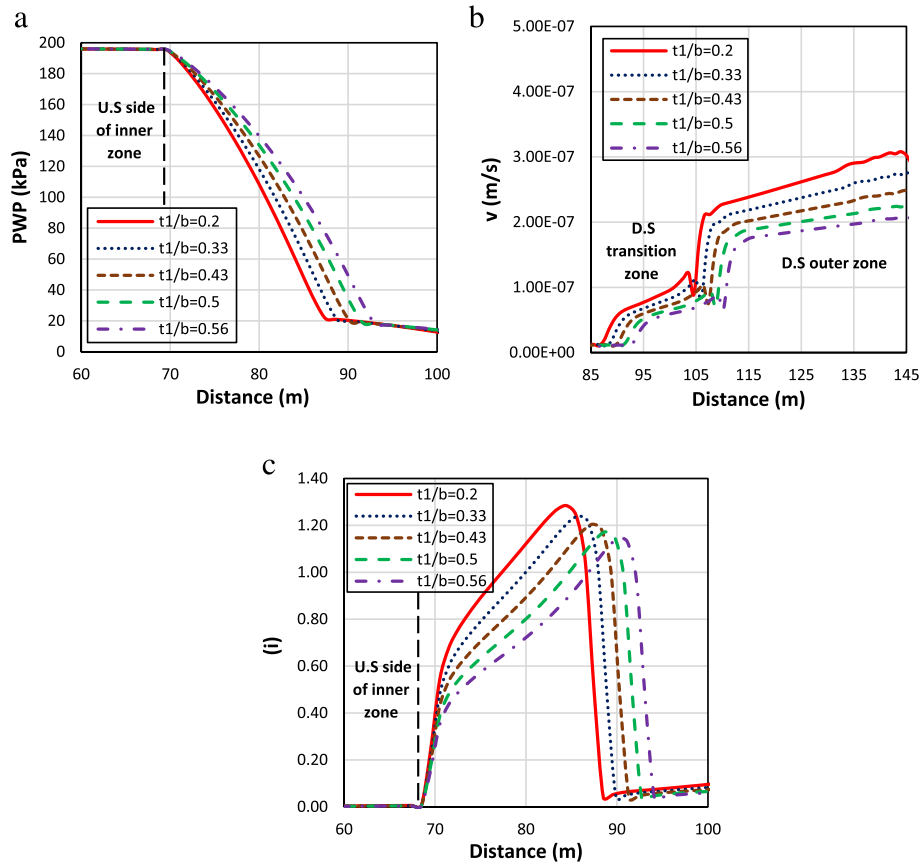


Figure 9-a illustrates the relation between the distance of the dam base and pore water pressure (PWP). From the figure, increasing the thickness of the inner zone causes an increase in the pore water pressure through the inner zone while it decreases the pore water pressure at the downstream side of the inner zone (By about 17%). This is due to the long path taken by the flow to reach the downstream side.

Figure 9-b shows the relation between the distance of the dam base and the velocity in the x-direction ( $v$ ). The figure illustrates that the velocity at the downstream outer zone decreases highly with the increase of the thickness of the inner zone (With about 28%). The relation between the distance of the dam base and the gradient in the x-direction ( $i$ ) is illustrated in Fig. 9-c. The figure shows that increasing the thickness of the inner zone decreases the gradient in the inner zone (With 11%). This may be due to increasing the length of flow which decreases the hydraulic slope.

**Effect of the thickness of the transition zones**

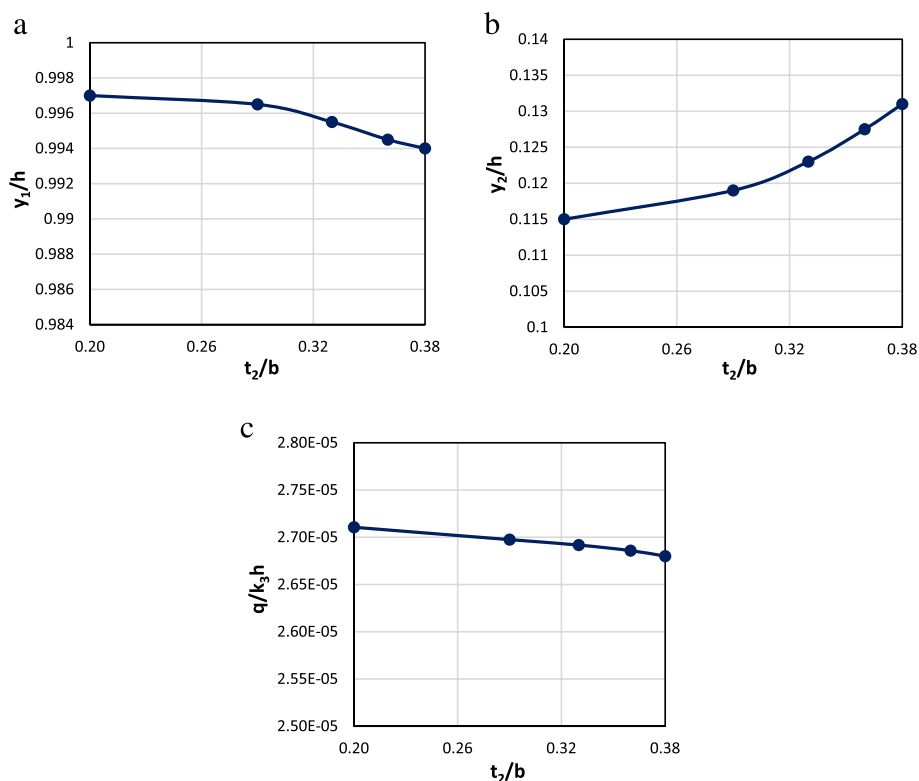
To investigate the effect of the thickness of the transition zones on the seepage through the zoned earth dam, five models are tested by increasing the top thickness ( $t_2$ ) from 1 to 5 times the control one.



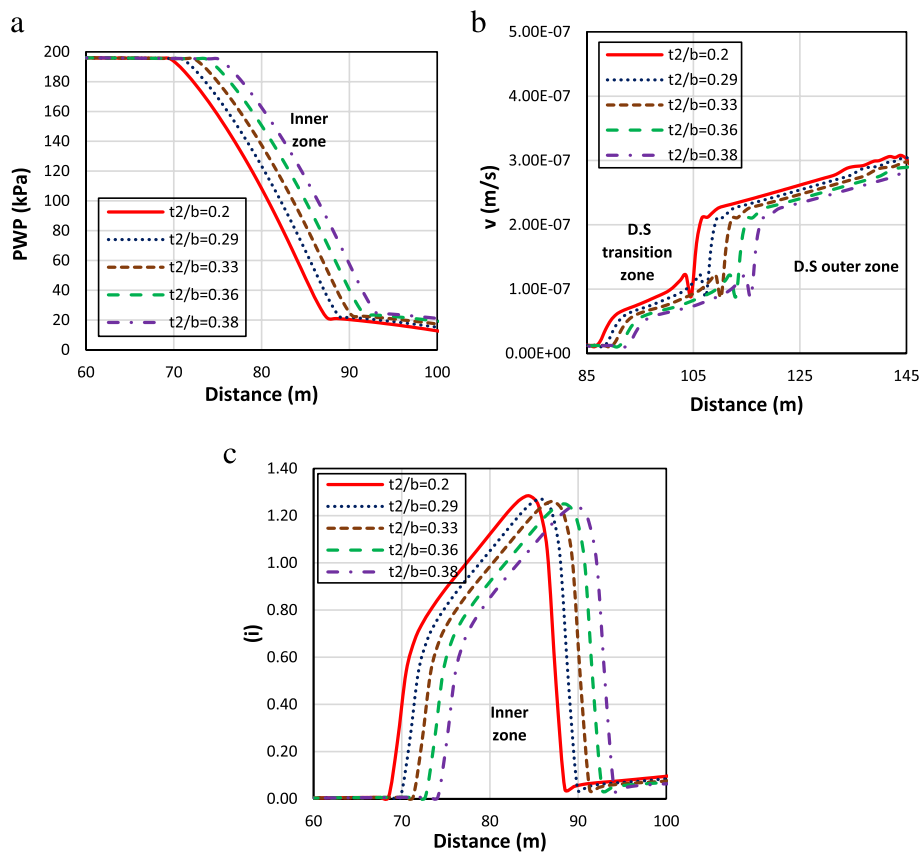
**Fig. 9** a Relation between the distance of the dam base and the pore water pressure for different  $t_1/b$ . b Relation between the distance of the dam base and the velocity in the x-direction for different  $t_1/b$ . c Relation between the distance of the dam base and the gradient in the x-direction for different  $t_1/b$

The relation between the relative thickness of the transition zone ( $t_2/b$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ) is shown in Fig. 10-a. The figure shows that increasing the thickness of the transition zones causes a very slight decrease in the phreatic line level at the upstream side of the inner zone. This is due to increasing the path taken by the flow through the upstream transition zone. Figure 10-b illustrates the relation between the relative thickness of the transition zone ( $t_2/b$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ), from which, increasing the thickness of the transition zones increases the phreatic line level at the downstream side of the inner zone (With 14%), as increasing the thickness of the downstream transition zone inhibits the flow which rises the phreatic line.

Figure 10-c shows the relation between the relative thickness of the transition zone ( $t_2/b$ ) and the seepage discharge ratio ( $q/k_3h$ ). It is noticed that the seepage discharge decreases slightly with increasing the thickness of the transition zones. The relation between the distance of the dam base and pore water pressure (PWP) is illustrated in Fig 11-a. From the figure, increasing the thickness of the transition zones increases the pore water pressure at the downstream side of the inner zone (By about 15%), as increasing the thickness of the downstream transition zone obstructs the flow which increases the pore water pressure.



**Fig. 10** **a** Relation between the relative thickness of the transition zone ( $t_2/b$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). **b** Relation between the relative thickness of the transition zone ( $t_2/b$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ). **c** Relation between the relative thickness of the transition zone ( $t_2/b$ ) and the discharge ratio ( $q/k_3h$ )



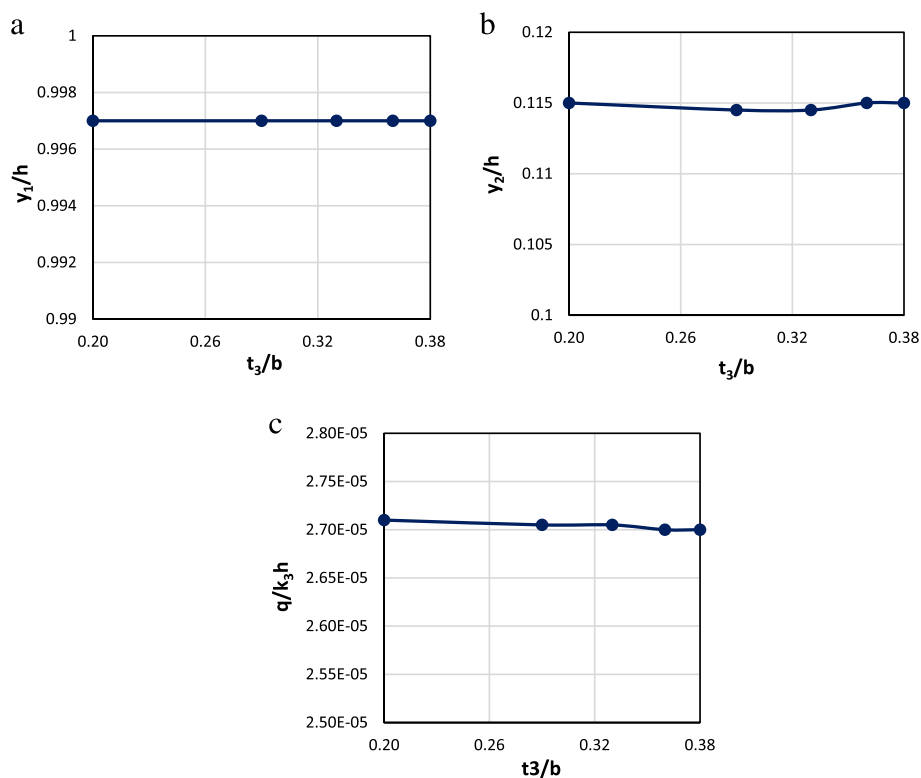
**Fig. 11** a Relation between the distance of the dam base and the pore water pressure for different  $t_2/b$ . b Relation between the distance of the dam base and the velocity in the x-direction for different  $t_2/b$ . c Relation between the distance of the dam base and the gradient in the x-direction for different  $t_2/b$

The relation between the distance of the dam base and the velocity in the x-direction ( $v$ ) is shown in Fig. 11-b. The figure shows that the velocity at the downstream outer zone decreases with the increase of the thickness of the transition zones. Figure 11-c illustrates the relation between the distance of the dam base and the gradient in the x-direction ( $i$ ). The figure shows that increasing the thickness of the transition zones decreases slightly the gradient in the inner zone.

**Effect of the thickness of the outer zones**

To know the effect of the thickness of the outer zones on the seepage through the zoned earth dam, five models are tested by increasing the top thickness ( $t_3$ ) from 1 to 5 times the control one.

Figure 12-a shows the relation between the relative thickness of the outer zone ( $t_3/b$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). The figure illustrates that increasing the thickness of the outer zones does not affect the phreatic line level at the upstream side of the inner zone. The relation between the relative thickness of the outer zone ( $t_3/b$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ) is illustrated in Fig. 12-b, from which, increasing the thickness of the outer zones does not affect the phreatic line level at the downstream side of the inner zone.



**Fig. 12** a Relation between the relative thickness of the outer zone ( $t_3/b$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). b Relation between the relative thickness of the outer zone ( $t_3/b$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ) c Relation between the relative thickness of the outer zone ( $t_3/b$ ) and the discharge ratio ( $q/k_3h$ )

The relation between the relative thickness of the outer zone ( $t_3/b$ ) and the seepage discharge ratio ( $q/k_3h$ ) is shown in Fig. 12-c. It is noticed that the seepage discharge decreases very slightly with increasing the thickness of the outer zones. Figure 13-a illustrates the relation between the distance of the dam base and pore water pressure (PWP). From the figure, increasing the thickness of the outer zones does not affect the pore water pressure at the downstream side of the inner zone.

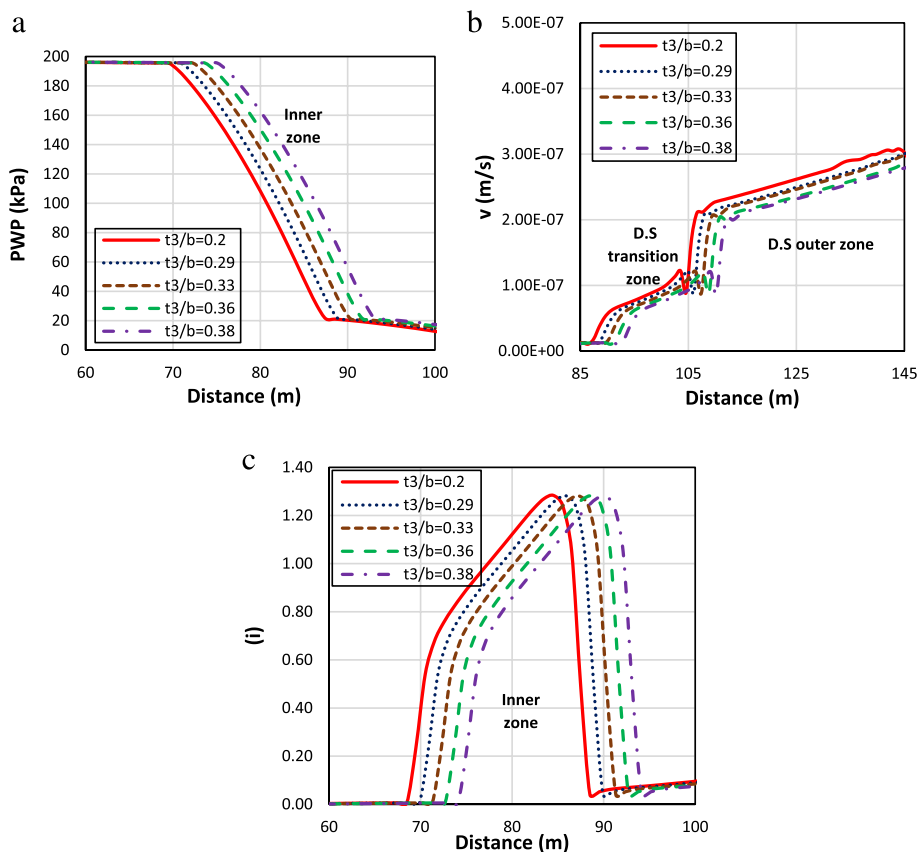
Figure 13-b shows the relation between the distance of the dam base and the velocity in the x-direction ( $v$ ). The figure illustrates that the velocity at the downstream outer zone decreases with the increase of the thickness of the outer zones (With about 12%). The relation between the distance of the dam base and the gradient in the x-direction ( $i$ ) is illustrated in Fig. 13-c. The figure shows that increasing the thickness of the outer zones decreases very slightly the gradient in the inner zone.

**Effect of the side slopes of the zones on the seepage through zoned earth dams**

**Effect of the side slopes of the inner zone (core)**

To determine the effect of the side slopes of the inner zone on the seepage through the zoned earth dam, five models are tested by increasing the side slopes ( $S_1$ ) from 1:3 to 1:1.

The relation between the side slopes of the inner zone ( $S_1$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ) is shown in Fig. 14-a. The figure shows that

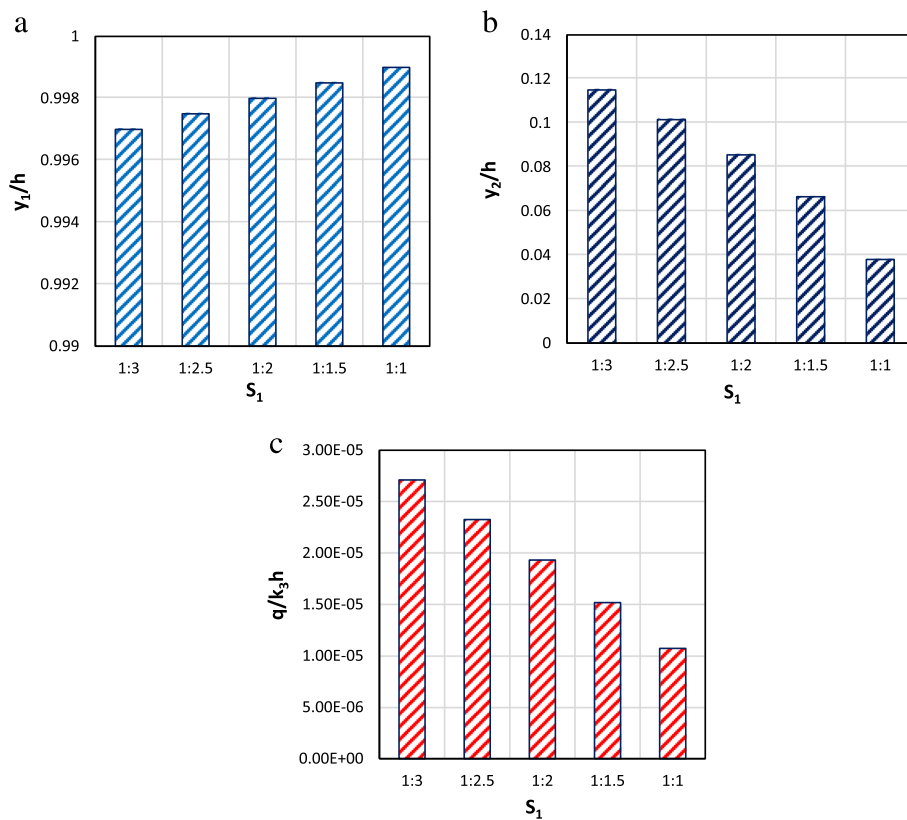


**Fig. 13** a Relation between the distance of the dam base and the pore water pressure for different  $t_3/b$ . b Relation between the distance of the dam base and the velocity in the x-direction for different  $t_3/b$ . c Relation between the distance of the dam base and the gradient in the x-direction for different  $t_3/b$

increasing the side slopes of the inner zone causes a slight increase in the phreatic line level at the upstream side of the inner zone, as increasing the side slopes of the inner zone causes more flow obstruction which rises the phreatic line at the upstream side. Figure 14-b illustrates the relation between the side slopes of the inner zone ( $S_1$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ), from which, increasing the side slopes of the inner zone decreases the phreatic line level at the downstream side of the inner zone (About 67%). This is due to the long path taken by the flow to reach the downstream side of the inner zone which causes more head loss.

Figure 14-c shows the relation between the side slopes of the inner zone ( $S_1$ ) and the seepage discharge ratio ( $q/k_3h$ ). It is concluded that the seepage discharge decreases significantly with increasing the side slopes of the inner zone (With about 153%). The relation between the distance of the dam base and pore water pressure (PWP) is illustrated in Fig. 15-a. From the figure, increasing the side slopes of the inner zone causes a decrease in the pore water pressure at the downstream side of the inner zone (By 68%). This may be due to increasing the distance taken by the flow to reach the downstream side.

The relation between the distance of the dam base and the velocity in the x-direction ( $v$ ) is shown in Fig. 15-b. The figure shows that the velocity at the downstream outer



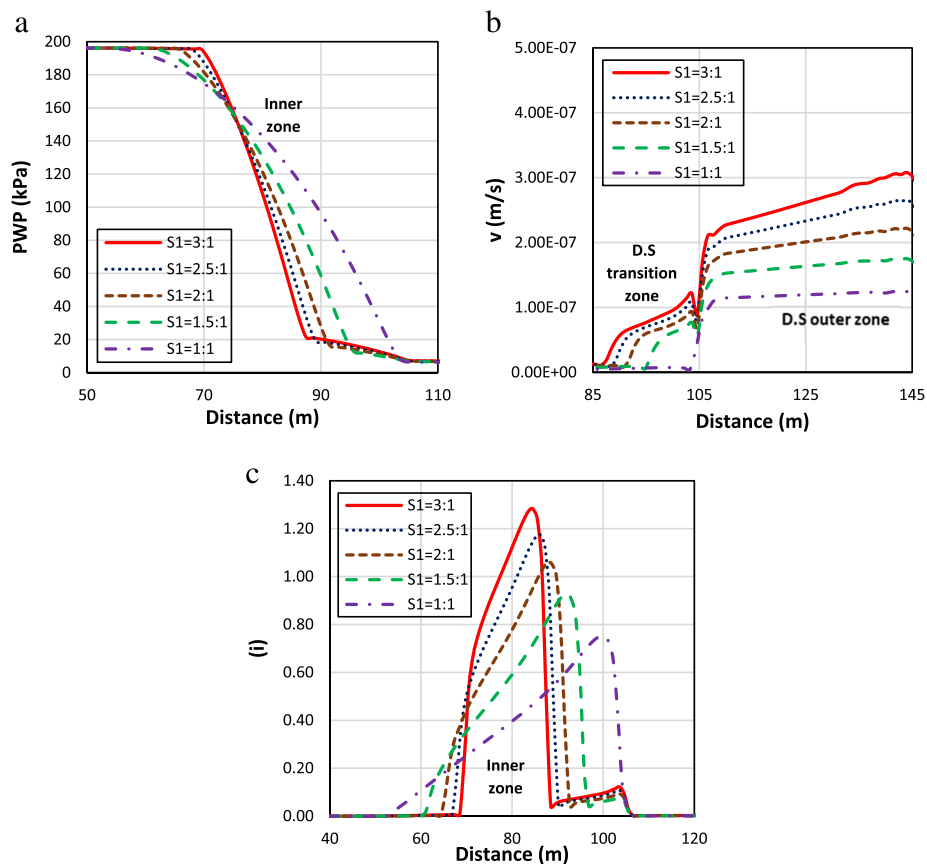
**Fig. 14** **a** Relation between the side slopes of the inner zone ( $S_1$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). **b** Relation between the side slopes of the inner zone ( $S_1$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ). **c** Relation between the side slopes of the inner zone ( $S_1$ ) and the discharge ratio ( $q/k_3h$ )

zone decreases with the increase of the side slopes of the inner zone (By about 60%). Figure 15-c illustrates the relation between the distance of the dam base and the gradient in the x-direction (i). The figure shows that increasing the side slopes of the inner zone decreases the gradient in the inner zone (With 42%), as the hydraulic slope decreases with the increase in the flow length.

**Effect of the side slopes of the transition zones**

To investigate the effect of the side slopes of the transition zones on the seepage through the zoned earth dam, five models are tested by increasing the side slopes ( $S_2$ ) from 1:1 to 3:1.

Figure 16-a shows the relation between the side slopes of the transition zones ( $S_2$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). The figure illustrates that increasing the side slopes of the transition zones causes a very slight decrease in the phreatic line level at the upstream side of the inner zone. This is due to increasing the distance taken by the water through the upstream transition zone. The relation between the side slopes of the transition zones ( $S_2$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ) is illustrated in Fig. 16-b, from which, increasing the side slopes of the transition zones increases highly the phreatic

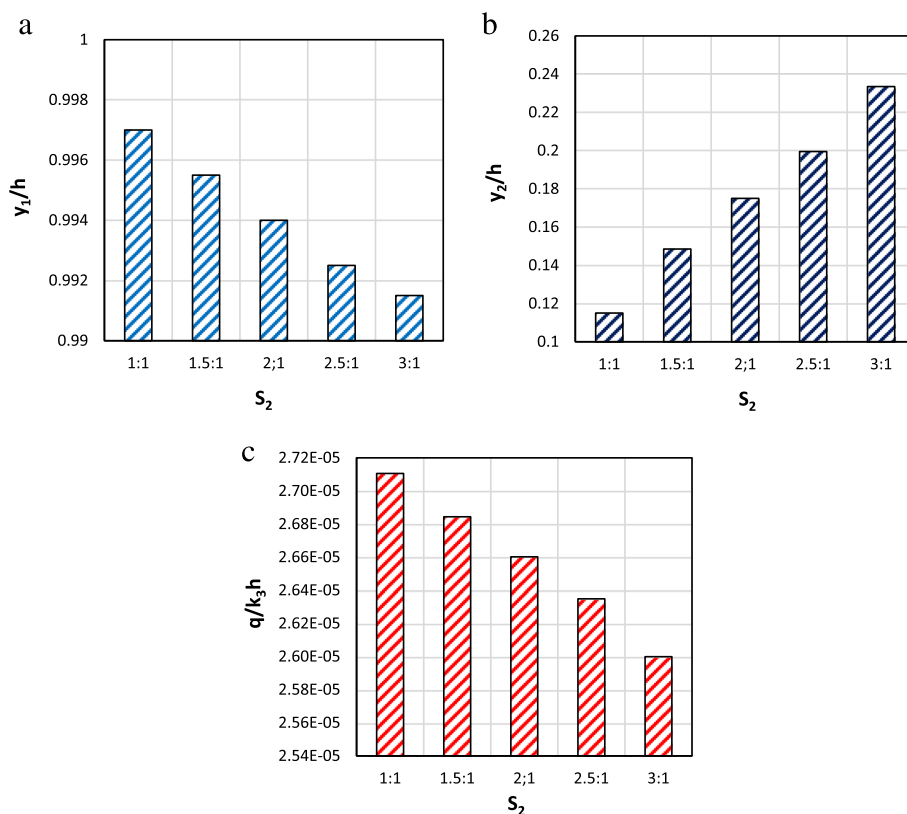


**Fig. 15** **a** Relation between the distance of the dam base and the pore water pressure for different  $S_1$ . **b** Relation between the distance of the dam base and the velocity in the x-direction for different  $S_1$ . **c** Relation between the distance of the dam base and the gradient in the x-direction for different  $S_1$

line level at the downstream side of the inner zone (With 103%), as increasing the side slope of the downstream transition zone obstructs the flow which increases the phreatic line.

The relation between the side slopes of the transition zones ( $S_2$ ) and the seepage discharge ratio ( $q/k_3h$ ) is shown in Fig. 16-c. It is seen that the seepage discharge decreases slightly with increasing the side slopes of the transition zones. Figure 17-a illustrates the relation between the distance of the dam base and pore water pressure (PWP). From the figure, increasing the side slopes of the transition zones causes an increase in the pore water pressure at the downstream side of the inner zone (With about 107%), as increasing the side slope of the downstream transition zone inhabits the flow which rises the pore water pressure.

Figure 17b shows the relation between the distance of the dam base and the velocity in the x-direction ( $v$ ). The figure illustrates that the velocity at the downstream transition zone decreases with the increase of the side slopes of the transition zones. The relation between the distance of the dam base and the gradient in the x-direction ( $i$ ) is illustrated in Fig. 17-c. The figure shows that increasing the side slopes of the transition zones decreases the gradient in the inner zone (With about 19%), while it causes an increase in the transition zone.



**Fig. 16** **a** Relation between the side slopes of the transition zones ( $S_2$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). **b** Relation between the side slopes of the transition zones ( $S_2$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ) **c** Relation between the side slopes of the transition zones ( $S_2$ ) and the discharge ratio ( $q/k_3h$ )

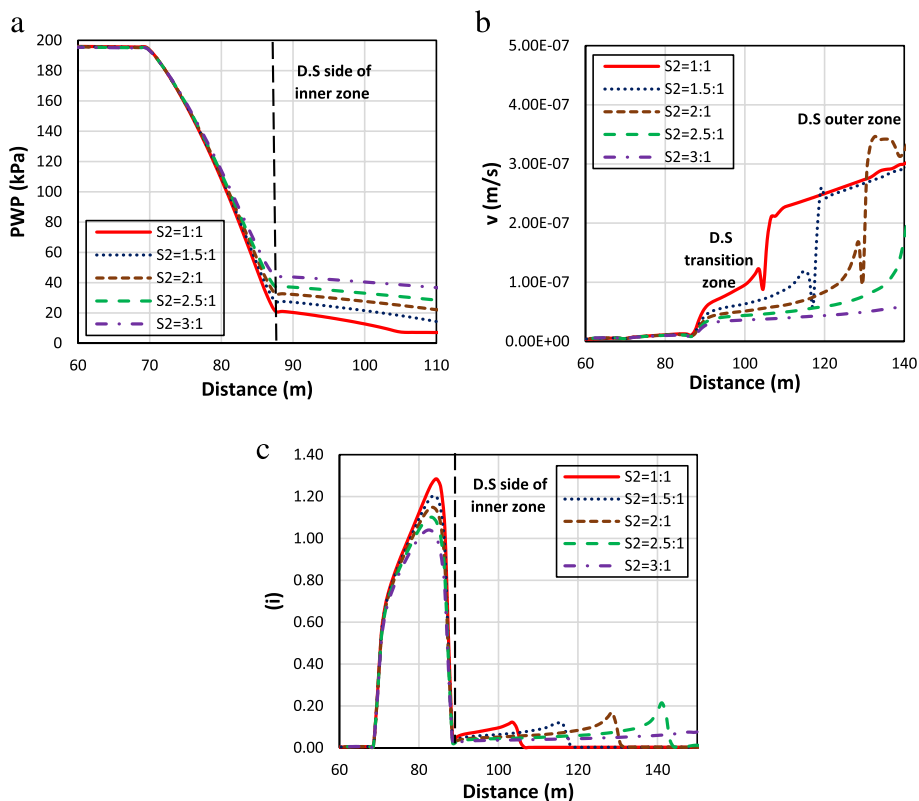
### Effect of the side slopes of the outer zones

To know the effect of the side slopes of the outer zones on the seepage through the zoned earth dam, five models are tested by increasing the side slopes ( $S_3$ ) from 3:1 to 5:1.

The relation between the side slopes of the outer zones ( $S_3$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ) is shown in Fig. 18-a. The figure shows that increasing the side slopes of the outer zones does not affect the phreatic line level at the upstream side of the inner zone. Figure 18-b illustrates the relation between the side slopes of the outer zones ( $S_3$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ), from which, increasing the side slopes of the outer zones causes an increase in the phreatic line level at the downstream side of the inner zone ((By about 30%), as increasing the side slope of the downstream outer zone causes more flow obstruction which rises the phreatic line.

Figure 18-c shows the relation between the side slopes of the outer zones ( $S_3$ ) and the seepage discharge ratio ( $q/k_3h$ ). It is seen that the seepage discharge decreases very slightly with increasing the side slopes of the outer zones. The relation between the distance of the dam base and pore water pressure (PWP) is illustrated in Fig. 19-a. From the figure, increasing the side slopes of the outer zones causes an increase in the pore water at the downstream side of the inner zone (With about 31%), as increasing





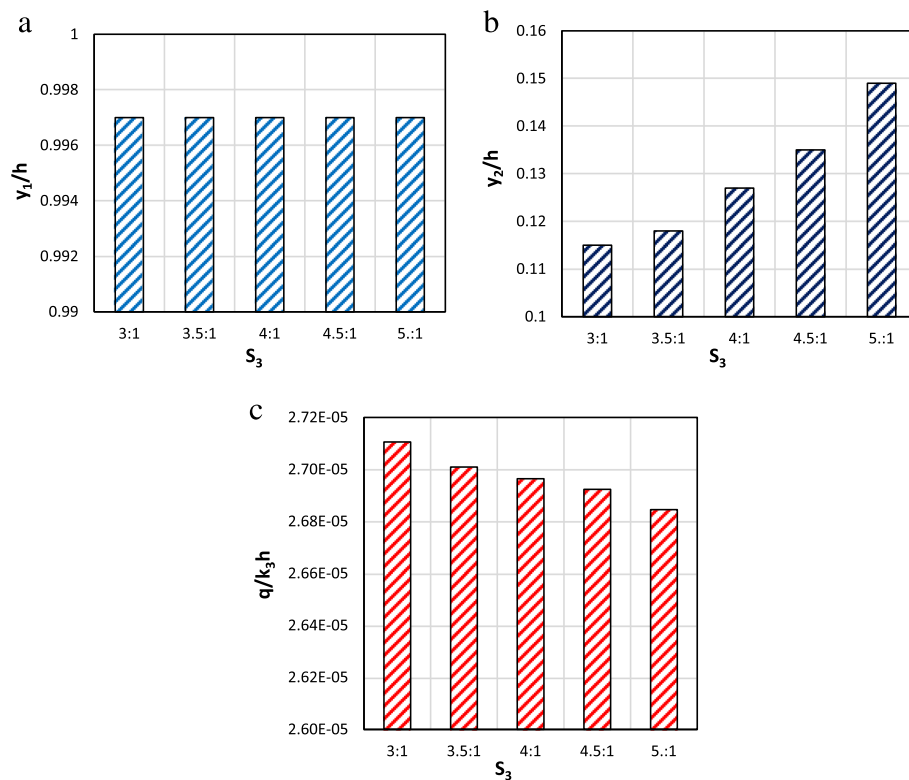
**Fig. 17** a Relation between the distance of the dam base and the pore water pressure for different  $S_2$ . b Relation between the distance of the dam base and the velocity in the x-direction for different  $S_2$ . c Relation between the distance of the dam base and the gradient in the x-direction for different  $S_2$

the side slope of the downstream outer zone inhibits the flow which increases the pore water pressure.

The relation between the distance of the dam base and the velocity in the x-direction ( $v$ ) is shown in Fig. 19-b. The figure shows that the exit velocity decreases with the increase of the side slopes of the outer zones (With 41%). Figure 19-c illustrates the relation between the distance of the dam base and the gradient in the x-direction ( $i$ ). The figure shows that increasing the side slopes of the outer zones causes a slight decrease in the gradient in the inner zone and decreases the exit gradient (About 41%).

### Selecting the optimum thickness ratio of the different zones

To determine the optimum thickness ratio ( $t/b$ ) of the different zones according to the minimum seepage flow depending on the most affected parameter, the cost of the used material is calculated as a function of its volume, by assuming other different parameters are constant. The symbol ( $a$ ) is a constant that represents the cost of the used material for each zone in case of minimum tested thickness ratio, which increases with the increase of the thickness of the different zones.



**Fig. 18** **a** Relation between the side slopes of the outer zones ( $S_3$ ) and the phreatic line level at the upstream side of the inner zone ( $y_1/h$ ). **b** Relation between the side slopes of the outer zones ( $S_3$ ) and the phreatic line level at the downstream side of the inner zone ( $y_2/h$ ). **c** Relation between the side slopes of outer zones ( $S_3$ ) and the discharge ratio ( $q/k_3h$ )

**Optimum thickness ratio of the inner zone (core)**

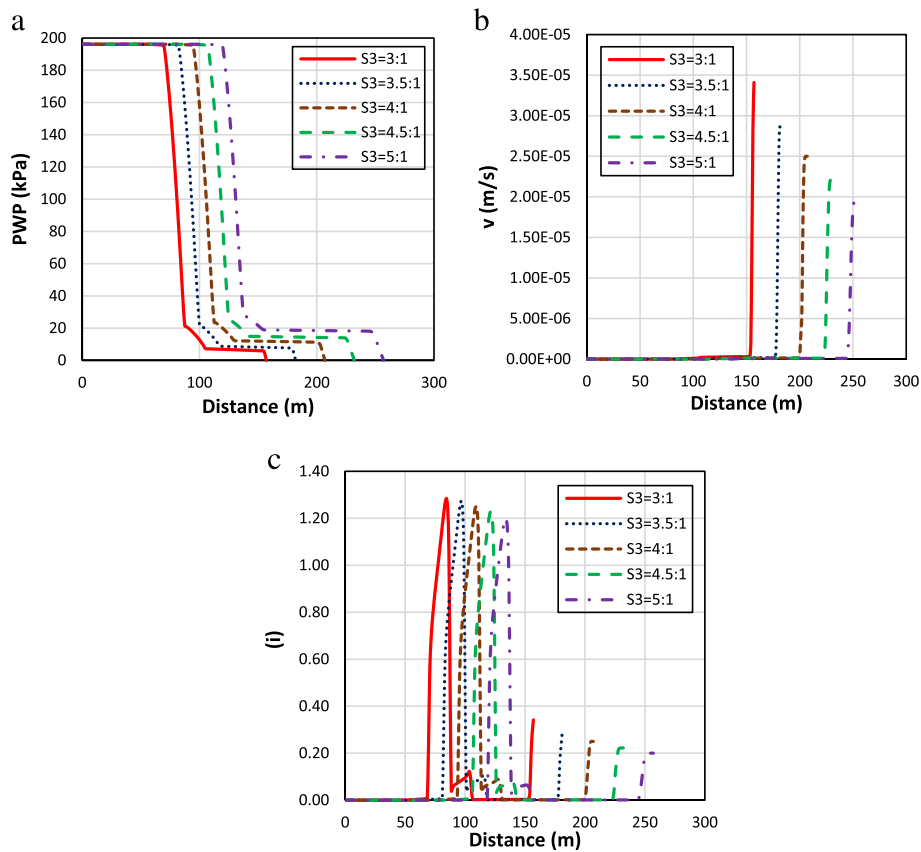
Figure 20 shows the relation between the relative seepage discharge and the cost of the used material of the inner zone against the different thickness ratios ( $t_1/b$ ). It is seen that the two curves representing the relative seepage discharge and the cost of the used material of the inner zone, intersect at a thickness ratio equal to 0.39 approximately. So, for a practical design, the reasonable optimum thickness ratio of the inner zone ( $t_1/b$ ) is about 0.4.

**Optimum thickness ratio of the transition zone**

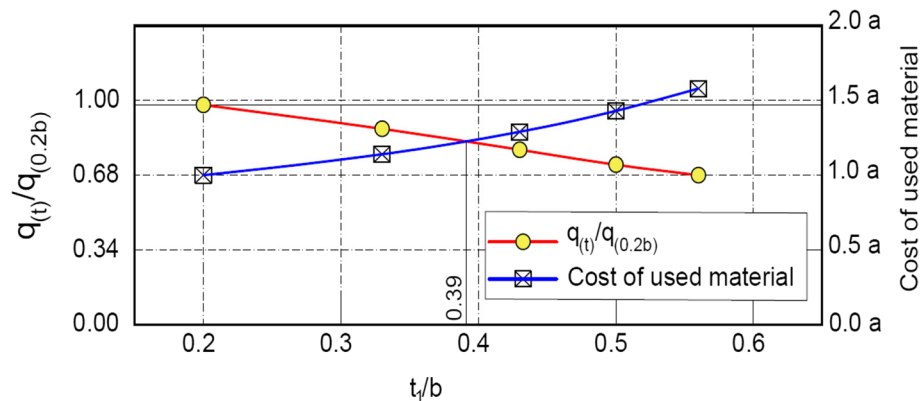
The relation between the relative water velocity and the cost of the used material of the transition zone against the different thickness ratios ( $t_2/b$ ) is shown in Fig. 21. The figure illustrates that the two curves representing the relative water velocity and the cost of the used material of the transition zone, intersect at a thickness ratio equal to 0.28. Then, the reasonable thickness ratio of the transition zone ( $t_2/b$ ) is 0.3.

**Optimum thickness ratio of the outer zone**

Figure 22 illustrates the relation between the relative water velocity and the cost of the used material of the outer zone against the different thickness ratios ( $t_3/b$ ). It is

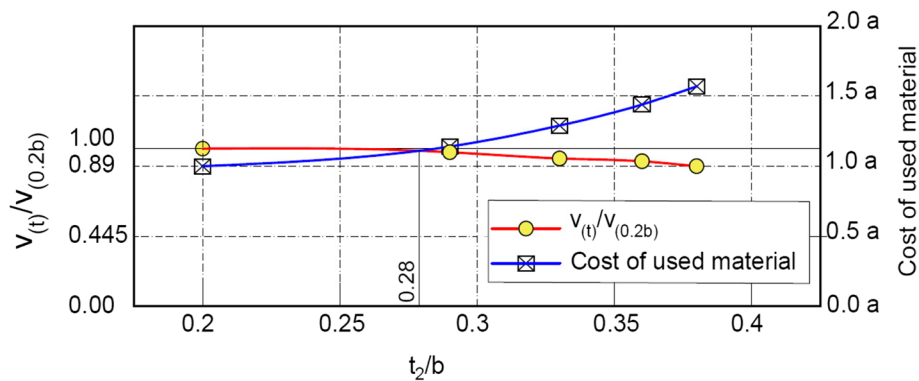


**Fig. 19** a Relation between the distance of the dam base and the pore water pressure for different  $S_3$ . b Relation between the distance of the dam base and the velocity in the x-direction for different  $S_3$ . c Relation between the distance of the dam base and the gradient in the x-direction for different  $S_3$

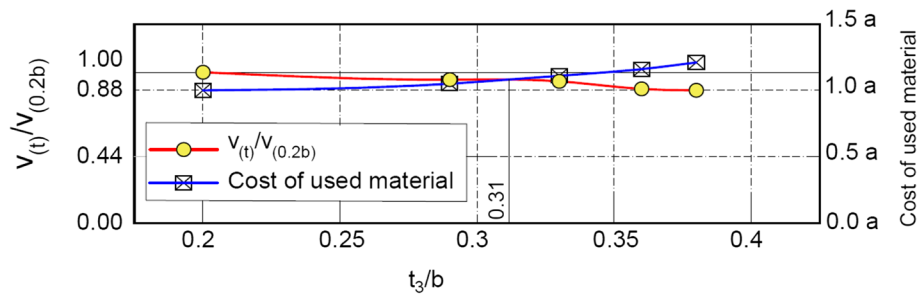


**Fig. 20** Relation between the relative seepage discharge, the cost of the used material, and the different thickness ratios of the inner zone ( $t_1/b$ )

noticed that the two curves representing the relative water velocity and the cost of the used material of the outer zone, intersect at a thickness ratio equal to 0.31 approximately. So, the optimum thickness ratio of the outer zone for a practical design is about 0.3.



**Fig. 21** Relation between the relative water velocity, the cost of the used material, and the different thickness ratios of the transition zone ( $t_2/b$ )



**Fig. 22** Relation between the relative water velocity, the cost of the used material, and the different thickness ratios of the outer zone ( $t_3/b$ )

**Selecting the optimum side slopes of the different zones**

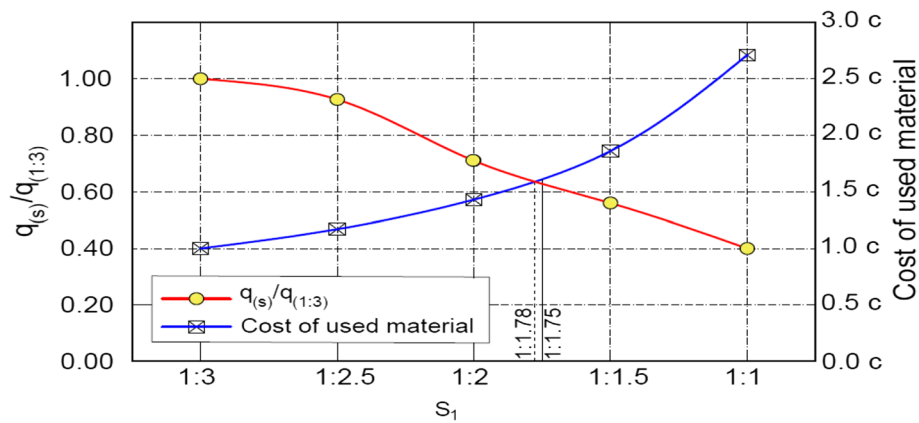
To investigate the optimum side slopes of the different zones according to the minimum seepage flow depending on the most affected parameter, the cost of the used material is calculated as a function of its volume, by assuming other different parameters are constant. The symbol (c) is a constant that represents the cost of the used material for each zone in case of minimum tested side slopes which increases with the increase of the side slopes of the different zones.

**Optimum side slopes of the inner zone (core)**

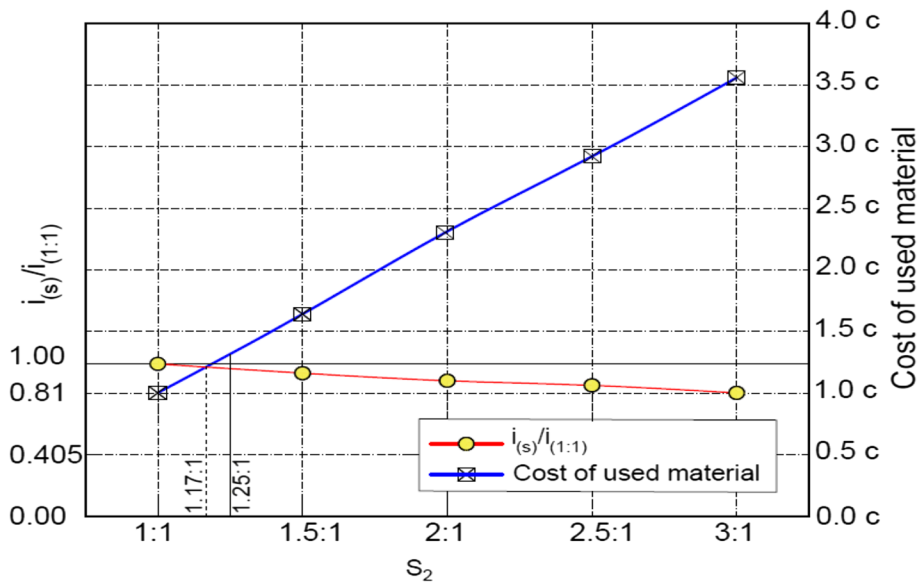
Figure 23 shows the relation between the relative seepage discharge and the cost of the used material of the inner zone against the different side slopes ( $S_1$ ). It is seen that the two curves representing the relative seepage discharge and the cost of the used material of the inner zone, intersect at side slopes equal to 1:1.78 approximately. So, for a practical design, the reasonable optimum side slope of the inner zone is about 1:1.75.

**Optimum side slopes of the transition zones**

The relation between the relative hydraulic gradient and the cost of the used material of the transition zones against the different side slopes ( $S_2$ ) is illustrated in Fig. 24. The figure shows that the two curves representing the relative hydraulic gradient and



**Fig. 23** Relation between the relative seepage discharge, the cost of the used material, and the different side slopes of the inner zone ( $S_1$ )

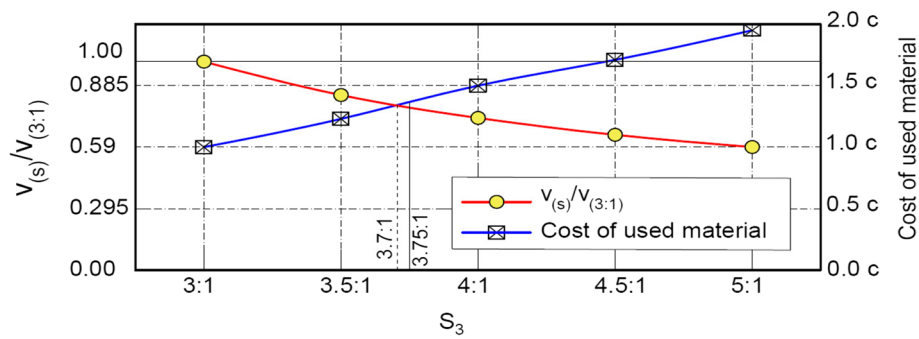


**Fig. 24** Relation between the relative hydraulic gradient, the cost of the used material, and the different side slopes of the transition zones ( $S_2$ )

the cost of the used material of the transition zones, intersect at side slopes equal to 1.17:1, this is because increasing the side slopes of the transition zones does not have a noticeable effect on the different seepage parameters, while it causes a remarkable increase of the volume of the used material. Then, the reasonable side slope of the transition zones is 1.25:1.

**Optimum side slopes of the outer zones**

Figure 25 illustrates the relation between the relative exit velocity and the cost of the used material of the outer zones against the different side slopes ( $S_3$ ). It is noticed that the two curves representing the relative exit velocity and the cost of the used material



**Fig. 25** Relation between the relative exit velocity, the cost of the used material, and the different side slopes of the outer zones ( $S_3$ )

of the outer zones, intersect at side slopes equal to 3.7:1 approximately. So, the optimum side slope of the outer zones for a practical design is about 3.75:1.

**Conclusions**

The present paper introduces a numerical analysis of seepage through zoned earth dams to achieve the most reasonable dimensions and geometry of the different zones according to the minimum seepage and cost of the used materials. The used Seep/w numerical model succeeds in investigating the seepage process through earth dams having an internal core and gives results in good agreement with the results obtained through the previous experimental and analytical studies. The best relative thickness of the inner, transition, and outer zones ( $t_1:t_2:t_3$ ) according to the minimum seepage and cost of the used materials is 2:1.5:1.5 respectively. The reasonable optimum side slopes (H:V) according to the minimum seepage and cost of the used materials of the inner, transition, and outer zones are 1:1.75, 1.25:1, and 3.75:1 respectively. Increasing the thickness of the inner zone decreases the pore water pressure, seepage discharge, and gradient through the dam body. A slight increase in the pore water pressure occurs by increasing the thickness of the transition zones, while a negligible effect is obtained for the outer zones. On the other hand, increasing the side slopes of the inner zone causes a significant decrease in the different seepage parameters. The pore water pressure increases by increasing the side slopes of the transition and outer zones, while a decrease is achieved in the velocity and gradient.

**Abbreviations**

- a Constant represents the cost of the used material for each zone in case of minimum tested thickness ratio (dimensionless).
- b Top width of the dam (m).
- c Constant represents the cost of the used material for each zone in case of minimum tested side slopes (dimensionless).
- h Upstream water head acting on the dam (m).
- i Hydraulic gradient (dimensionless).
- $k_1$  Hydraulic conductivity of the inner zone (m/s).
- $k_2$  Hydraulic conductivity of the transition zones (m/s).
- $k_3$  Hydraulic conductivity of the outer zones (m/s).
- $k_c$  Permeability coefficient of the core (m/s).
- $k_d$  Permeability coefficient of the dam body (m/s).
- PWP Pore water pressure (kPa).
- Q Discharge of dam without core per unit length ( $m^2/s$ ).
- $Q_1$  Discharge of dam with core per unit length ( $m^2/s$ ).

$q$	Seepage discharge rate per unit length ( $m^2/s$ ).
$S_1$	Side slopes of the inner zone (dimensionless).
$S_2$	Side slopes of the transition zones (dimensionless).
$S_3$	Side slopes of the outer zones (dimensionless).
$t_1$	Top thickness of the inner zone (m).
$t_2$	Top thickness of the transition zone (m).
$t_3$	Top thickness of the outer zone (m).
$v$	Water velocity in the x-direction (m/s).
$y_1$	Phreatic level at the upstream side of the inner zone, taking the dam base as the datum (m).
$y_2$	Phreatic level at the downstream side of the inner zone, taking the dam base as the datum (m).

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**Authors' contributions**

MM: Investigation, Methodology, Formal Analysis, Writing – original draft. SZ: Supervision, Conceptualization, Writing – review & editing. The authors read and approved the final manuscript.

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**Declarations****Competing interests**

The authors declare that they have no competing interests.

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**References**

- Arora K R (2001) Irrigation, Water Power and Water Resources Engineering (in SI Units). standard publishers
- Garg S K (2013) Irrigation engineering and hydraulic structures. 23rd rev. ed., Khanna Publishers, New Delhi, India
- Zhang LM, Xu Y, Jia JS (2009) Analysis of earth dam failures: A database approach. *Georisk* 3(3):184–189. <https://doi.org/10.1080/17499510902831759>
- Flores-Berrones R, Lopez-Acosta N P (2011) Internal erosion due to water flow through earth dams and earth structures. *Intech open*, pp. 284–294. <https://doi.org/10.5772/24615>
- Boroomand MR, Mohammadi A (2019) Evaluation of earth dam leakage considering the uncertainty in soil hydraulic parameters. *Civil Eng J* 5(7):1543–1556. <https://doi.org/10.28991/cej-2019-03091351>
- Sazzad MM, Islam MM (2019) A comprehensive study of different types of seepage control measures for earth dam using FEM. *J Civ Construction Eng* 5(1):24–37. <https://doi.org/10.5281/zenodo.2610954>
- Galindo R, Sánchez-Martín J, Olalla Marañón C (2020) Sustainable Construction of Earth Dams: Use of Heterogeneous Material from the Dam Site. *Sustainability* 12(23):9940. <https://doi.org/10.3390/su12239940>
- Sazzad M M, Alam S (2020) Numerical investigation of seepage through earth dam by FEM. In 5th International Conference on Advances in Civil Engineering (ICACE), Bangladesh
- Fattah MY, Al-Labban SN, Salman FA (2014) Seepage analysis of a zoned earth dam by finite elements. *Int J Civil Eng Technol (IJCIET)* 5(8):128–139
- Sazzad M M, Roy M, Rahman M S (2014) Comparison between numerical and analytical solution of seepage flow through earth dam. In 2nd International Conference on Advances in Civil Engineering, pp. 300–306
- Fakhari A, Ghanbari A (2013) A simple method for calculating the seepage from earth dams with clay core. *J Geoenvironmental Eng* 8(1):27–32. [https://doi.org/10.6310/jog.2013.8\(1\).4](https://doi.org/10.6310/jog.2013.8(1).4)
- Sazzad MM, Roy MRINMOY, Rahman MM (2015) FEM based seepage analysis through earth dam. *Int J Adv Struct Geotechnical Eng* 4(3):158–164
- Hassan WH (2015) Application of a genetic algorithm for the optimization of a cut-off wall under hydraulic structures. *J Appl Water Eng Res* 5(1):22–30. <https://doi.org/10.1080/23249676.2015.1105161>
- Khassaf SI, Madhlom AM (2017) Effect of impervious core on seepage through zoned earth dam (case study: Khassa Chai dam). *Int J Sci Eng Res* 8(2):1053–1064
- Mazaheri A, Komasi M, Mohammadzadeh A, Beiranvand B (2019) Optimization of horizontal drain dimensions in heterogeneous earth dams using Artificial Neural Network (ANN): A case study on Marvak dam. *J Appl Res Water and Wastewater* 6(2):109–116. <https://doi.org/10.22126/arww.2019.1403>
- Salem MN, Eldeeb HM, Nofal SA (2019) Analysis of seepage through earth dams with internal core. *Int J Eng Res* 8(8):768–777. <https://doi.org/10.17577/IJERTV8IS080168>
- Hassan WH (2019) Application of a genetic algorithm for the optimization of a location and inclination angle of a cut-off wall for anisotropic foundations under hydraulic structures. *Geotech Geol Eng* 37(2):883–895. <https://doi.org/10.1007/s10706-018-0658-9>

18. Komasi M, Mohammadzadeh A, Beiranvand B (2020) Optimization of horizontal drain dimensions in homogeneous earth dams using neural network. *Int J Min Geo-Eng* 54(2):173–177. <https://doi.org/10.22059/ijmge.2019.278411.594792>
19. Mostafa M M, Shen Z (2021) A review on analysis of seepage in zoned earth dams. 2<sup>nd</sup> International Conference on Civil Engineering: Recent Applications and Future Challenges, Hurghada, Egypt, 2 (1) 137-146
20. Abdel-Kawy AO, Aboulatta NM, Elmolla DA (2021) Effects of core characteristics on seepage through earth dams. *Water Pract Technol* 16(4):1248–1264. <https://doi.org/10.2166/wpt.2021.053>
21. Alzamily ZN, Abed BS (2021) Experimental and theoretical investigations of seepage reduction through zoned earth dam material with special core. *Mater Today Proc* 61(551):998–1005. <https://doi.org/10.1016/j.matpr.2021.10.283>
22. Koś K, Gruchot A, Zawisza E (2021) Bottom Sediments from a Dam Reservoir as a Core in Embankments - Filtration and Stability: A Case Study. *Sustainability* 13(3):1221. <https://doi.org/10.3390/su13031221>
23. Beiranvand B, Rajaei T (2022) Application of artificial intelligence-based single and hybrid models in predicting seepage and pore water pressure of dams: A state-of-the-art review. *Adv Eng Softw* 173:103268. <https://doi.org/10.1016/j.advengsoft.2022.103268>
24. Hassan WH, Hussein HH, Khashan DH, Nile BK (2023) Optimum hydraulic design of cut-off under hydraulic structures using the simulation–optimization method. *Model Earth Syst Environ* 9(1):493–505. <https://doi.org/10.1007/s40808-022-01511-6>
25. GEO-SLOPE (2018) Seepage Modeling with SEEP/W – An Engineering Methodology (2018) Edition. GEO-SLOPE International. Available from: <http://www.geo-slope.com>
26. Kachare J, Jagtap S (2017) Seepage analysis of Gangapur, the earthen dam using Geo-Studio software. *Int J Recent Adv Eng Technol* 5(3):29–36
27. USBR Design Standards (2012) No. 13. Embankment Dams. U.S. Department of the Interior Bureau of Reclamation
28. Rozanov N (1978) *Hydrotechnical Structures*. Strouizdat, Moscow, pp 168–173
29. Rozanov N (1983) *Earth Dams*. Strouizdat, Moscow, pp 118–123
30. Rezk MA (1995) Earth dam with an internal core. *Alexandria Eng J Alexandria Univ* 34(1):C1–C13
31. Rezk MA, Senoon AE (2011) Analytical solution of seepage through earth dam with an internal core. *Alexandria Eng J, Alexandria Univ* 50(1):111–115. <https://doi.org/10.1016/j.aej.2010.10.001>

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