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# Scour depth downstream sharp-crested weir



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## Abstract

Weirs are hydraulic obstructions constructed across an open-channel flow causing a rise in water which is useful to convert water to the channels upstream as well as using these structures as a measuring device. There are different types, and a sharp-crested weir is one of them. Local scour downstream hydraulic structures are one of the most critical problems affecting their safety and stability. Many studies conducted to prevent these facilities from collapsing, but the effect of adding a solid apron downstream weir still need deep studies. The current study included a survey of scour reduction downstream sharp-crested weir using a solid apron consisting of three different lengths, four different discharges, and sand bed material to calculate the maximum depth and length of scour hole when using an apron solid. The results proved that the depth and length of the scour hole are reduced to 19.3% and 27.7%, respectively, when an apron solid is used. Two empirical equations were found to determine the maximum depth and length of the hole with a coefficient of determination are 0.892 and 0.804, respectively.

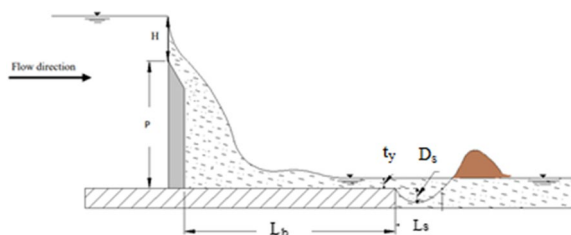
**Keywords:** Scour, Local scour, Sharp-crested weir, Solid apron, Weir, Open channel hydraulic

## Introduction

The aim of building dams across the river is not only for storage but also to control the water level in the river and raise its level. The water falling from the top of the dams may cause these dams to fail and collapse due to scour; to avoid this problem, many studies have been conducted that work to reduce scour behind these facilities. Hamidifar et al. [1] studied scour downstream channels using an apron solid with a length of 1 m consisting of two rough and smooth surfaces for comparison, using five different sizes of coarse protrusions  $K_s$  and five different sizes of bed materials. The results showed that the greater the roughness of the apron, the more energy dissipated increases, and thus, the scour decreases as the resistance to flow increases due to the roughness. A lot of flow energy is dissipated compared to if a smooth apron solid is used. Alwan et al. [2] studied local scour formed by the V-notch using four different models of apron solid made of wood with four different angles for each model:  $0^\circ$ ,  $2.73^\circ$ ,  $5.19^\circ$ , and  $7.43^\circ$ . Design pump discharge of 40 l/s and a sluice gate were placed at the end of the channel to regulate the depth of the tailwater. The findings show that the dimensions of the borehole in the solid of the straight apron are more relevant than the inclined floors, and the depth of the scour downstream apron solid expands with the slope of the bed. Kumar and Ahmad [3]

conducted a laboratory study of scour downstream piano key weir type A, which consists of three cycles. Using two different discharges and different depths of the tailwater, the experiments were conducted using an apron with a length of 20 cm and without an apron solid. They conclude that the dimensions of the scour hole are reduced when using an apron solid under dams compared to if an apron solid was not used. The scour depth reaches 80% of the height of the weir in the absence of an apron solid. Elnikhely and Fathy [4] studied scour downstream of a triangular labyrinth weir using an artificial channel for five different discharges ranging from 3 to 10 l/s at four different angles of the weirs: 45°, 60°, 75°, and 90°. The length of the apron solid is 1.2 m made from granite material to avoid soil deformation under the influence of water. The results show that the best angle for triangular labyrinth weirs, which gives the least scour downstream of the weir, is 60°. This angle also provides the most significant energy loss compared to the linear weir. The stilling basins of the short type dissipate the flow energy insufficiently, which causes the river bottom to deviate. Rumann et al. [5] assessed the impact of the projected fortifications' energy loss on the development of scours in the Hričov weir's downstream riverbed. At the hydraulic laboratory, a three-dimensional physical model was created using Froude's mechanical similarity criteria. For flood flow scenarios, simulations of both the weir's symmetric and asymmetric operations were performed. This technique provided a quick, dependable, and accurate way to examine riverbed deformations on physical models. The suggested fortification greatly reduced the scouring in the riverbed allowing symmetric operation even during flood flow, according to simulations and later observations. The size of the scours has shrunk by 50% from the state it was in before. Obaida [6] studied reducing the scour downstream piano key weir type C by using two distances for the apron solid with a length of 90 and 120 cm. The results of the study showed that the apron solid with a length of 120 cm works to reduce the scour compared to an apron solid with a length of 90 cm. Lantz et al. [7] conducted a laboratory study on reducing the dimensions of scour hole of the resulting piano key weir using three different lengths of apron solid and cutoff. Also, two experiments were conducted without using an apron solid and a partition wall for comparison using bed materials consisting of regular gravel. The study's results proved that using an apron solid with a length of 1 P, 1.5 P, and 2 P from the depth of weir, P reduces the dimensions of scour hole by 61%, 75%, and 83%, respectively. The scour depth also decreases by 8% when the length of the apron solid is increased from 1.5 P to 2 P from the height of the weir. In terms of economics and cost, the size of the protective floor 1.5 P proved highly efficient in reducing the piano key weir scour. Kumar and Ahmad [8] studied the scour downstream piano key weir, which consists of three cycles with and without apron solid under free and submerged flow conditions and for different tail water depths and discharges. Three distances were used for the apron solid. The results of the study showed that the use of an apron solid reduces the depth of scour by 60–80%.

This study aimed to reduce scour downstream of hydraulic structures which caused many dangerous failures in these structures using a water jet falling on a concrete apron instead of a movable bed as well as choosing the best length which causes minimum protection work required downstream hydraulic structures using three different downstream distances.



**Fig. 1** Definition parameter of weir and channel

**Dimensional analysis**

The parameters that affected the flow over the sharp-crested weir are  $D_s$  and  $L_s$ =peak height and distance of the scour hole  $L$ ;  $Q$ =discharge  $L^3/T$ ;  $H$ =head of water above the sharp-crested weir  $L$ ;  $W$ =width of the channel  $L$ ;  $L_b$ =length of apron solid  $L$ ;  $Y_t$ =tailwater depth  $L$ ;  $g$ =gravitational acceleration  $L/T^2$ ;  $P$ =weir depth  $L$ ;  $D_{50}$ =medium grain size diameter of the soil  $L$ ;  $\mu$  = dynamic viscosity of water  $M/L.T$ ;  $\rho$  = density of water  $M/L^3$ ; and  $\rho_s$  = density of the sediment  $M/L^3$ . Given the importance of knowing the effect of using three different lengths for the apron solid in maintaining the safety of hydraulic structures from collapse, the most important factors affecting the size of the borehole were identified under the same parameters of the study (Fig. 1). Then, the following equation can be written as follows:

$$D_s = f_{1,1}(Q, H, L_b, Y_t, g, P, D_{50}, \mu, \rho, \rho_s) \tag{1.1}$$

$$L_s = f_{1,2}(Q, H, L_b, Y_t, g, P, D_{50}, \mu, \rho, \rho_s) \tag{1.2}$$

Since the width of channel  $W$ , medium grain size diameter  $D_{50}$  is located at the bottom of the channel and  $\rho_s, \rho$  is a constant value, and then their effect can be canceled. The laboratory temperature was also imposed at 15 °C, a constant value. Therefore, the viscosity  $\mu$  will also have a fixed value in open channels; the value of Reynolds number  $Re$  is significant, so the fluid behaves according to the theory of non-viscosity; as the flow reaches complete turbulence, then it can neglected its effect [9]. During the experiments, Eqs. (1.1) and (1.2) become as follows:

$$D_s = f_{2,1}(Q, H, L_b, Y_t, g) \tag{2.1}$$

$$L_s = f_{2,2}(Q, H, L_b, Y_t, g) \tag{2.2}$$

“Buckingham  $\pi$  theorem” can be applied to Eqs. (2.1) and (2.2), and then, this equation can be written as follows:

$$\frac{D_s}{Y_t} = f\left(F_r, \frac{L_b}{Y_t}, \frac{H}{Y_t}\right) \tag{3.1}$$

$$\frac{L_s}{Y_t} = f\left(F_r, \frac{L_b}{Y_t}, \frac{H}{Y_t}\right) \tag{3.3}$$

$$Fr = \frac{V}{\sqrt{gY_t}} \quad (4)$$

where:

*Fr*: Froude number.

*V*: Velocity of flow *L/T*.

## Experimental setup

### Laboratory channel

The experiments were conducted in a concrete channel of 25 m in length, 0.81 m in width, and 0.76 m in height at the hydraulic laboratory of the University of Mosul (Fig. 2). A point gauge with an accuracy of  $\pm 0.1$  mm was used to measure water levels. The water is pumped from a ground tank below the laboratory floor with a closed flow system with a maximum discharge of 60 l/s [10]. A weir was installed at the end of the channel to measure the discharge of the channel using the height-discharge calculation, volumetric method (measuring volume of water — constant volume — at a certain time).

A sharp-crested weir was made of glass plastic, 4 cm thick, 81 cm wide, and 25 cm highest, according to British specifications [11] (Fig. 2).

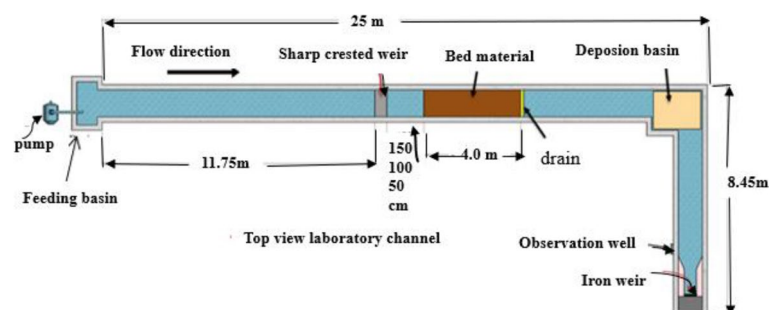
The weir is installed at a distance of 1175 cm from upstream. The apron solid is fixed downstream of the weir with a distance of 50, 100, and 150 cm.

### Bed material used in laboratory experiments

The sand is use as a material for the sedimentary bed to measure the depth and length of the scour hole downstream of the apron solid for the sharp-crested weir. A uniform sample is taken of sand passing through the sieve with an average opening diameter of 2 mm and the remainder on the sieve with an average aperture diameter of 0.5 mm. After conducting the sieve analysis, the medium grain size diameter of the sand  $D_{50}$  was used equal to 0.5 mm, and the standard deviation  $\sigma$  equal to 1.3. Sand is considered uniform if it is  $\sigma \leq 1.4$  [12]. The study area of 4 m is spread with sand of 20 cm thickness as the bed material of the channel used in the experiments downstream of the apron solid for the weir.

### The mechanism of conducting laboratory experiments

A sharp-crested weir was installed at the bottom of the concrete channel with different distances above the floor of the concrete channels 50, 100, and 150 cm, considered



**Fig. 2** Details of the channel laboratory

an apron solid. A layer of sand was placed at the bottom of the channel with a length of 4 m and a thickness of 20 cm. The surface of the sand was leveled with the surface of the solid apron. Two-point gauges were used; the first zeroed at the crested weir to measure the head of water after reaching a steady flow state. The second one moved on a rail and zeroed on the surface of the sand and channel bed used to measure the scour hole depth and length and the tailwater depth downstream the sharp-crested weir. The scour hole dimensions were taken after a certain period, approximately 3 h from the start of the operation, to make sure that the scour process stopped and the movement of the grain material stopped; the dimensions of the scour hole were measured downstream the weir after making sure that the water which passes over the sand layer was drained, as the point gauge was moved every 5 cm for the entire width and length of the study area, according to the preliminary experiments that were conducted for a week, and previous studies used the distance of the same grid [13]. After completing the readings, the bed materials were mixed well and then refurnished and leveled again, and the rest of the discharge was used for the exact first distance of the apron solid, and the discharge was reused for the second and third distances of the apron solid.

## Results and discussion

The effect of nondimensional variables on the maximum depth and length of the scour hole was studied when changing the size of the apron solid by using the theory of dimensional analysis. The statistical program SPSS 21 was used to find empirical equations of the depth and length of scour hole in terms of the influencing factors [14]. The 3D model of the sedimentary bed was drawn using the program Surfer-13 to show the scour areas downstream weir.

### Froude number ( $Fr$ ) effect

Froude number  $Fr$  is an essential factor influencing the scour hole's characteristics (scour depth and downstream of a fixed bed) such as scour around a pier [15–17]. The results showed that there is a direct relationship between the relative depth and length of scour hole to the height of tailwater  $D_s/Y_t$  and  $L_s/Y_t$  with Froude number  $Fr$ . When the  $Fr$  increased, the dimension of scour hole increases (Table 1 and Figs. 3 and 4).

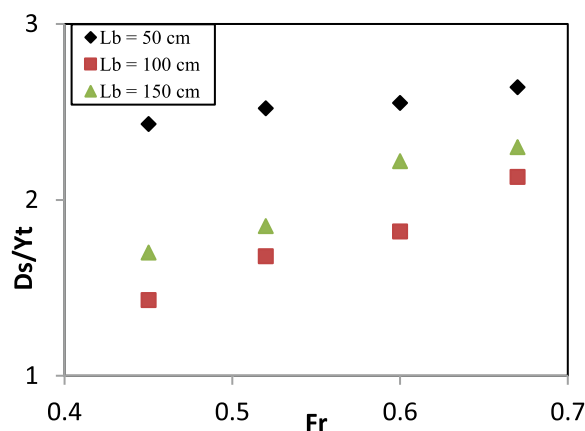
From the figures, it can be seen that when the apron solid length is 50 cm, the depth and length of scour hole increase compared with the lengths of 100 and 150 cm; because the flow is more turbulent when it reaches the sand layer, the long distance assists to reduce turbulence and calming the flow [3] and [13]. The dimension of the scours hole increases at the apron solid 150 cm compared to 100 cm because of forming vortices downstream structures, and these vortices increase when the approach apron increases and leads to increase scour dimensions.

### Effect length of apron solid ( $L_b$ )

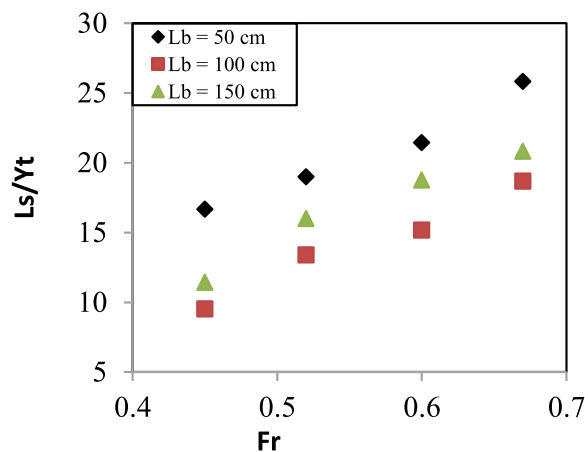
A hydraulic apron solid was used to reduce the dimensions of scour hole; the results showed that increasing the length of the apron solid by 50 to 100 cm reduces the flow energy before reaching the study area for the same discharge. The study's results showed that the highest percentage decrease in the depth and length of scour hole was 19.3% and 27.7%, respectively, at the discharge of 25 l/s because the area on which the water

**Table 1** The laboratory measurements as well as scour hole depth and length calculation of a sharp-crested weir at apron solid lengths 50, 100, and 150 cm for weir heights 25 cm

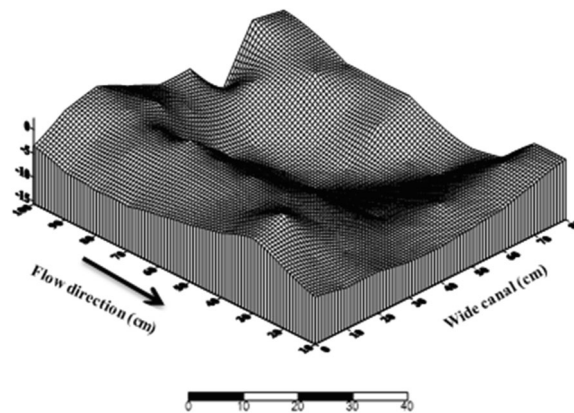
$H$ (cm)	$Y_t$ (cm)	$L_b$ (cm)	$D_s$ (cm)	$L_s$ (cm)	$D_s/Y_t$	$L_b/Y_t$	$H/Y_t$	$L_s/Y_t$	$Q$ (l/s)	$Fr$
4.5	4.2	50	10.20	70	2.43	12.0	1.07	16.67	10	0.45
6.3	5.0	50	12.60	95	2.52	10.0	1.26	19.00	15	0.52
7.0	5.6	50	14.28	120	2.55	9.0	1.25	21.43	20	0.60
8.5	6.0	50	15.84	155	2.64	8.3	1.41	25.83	25	0.67
4.5	4.2	100	6.00	40	1.43	23.8	1.07	9.52	10	0.45
6.3	5.0	100	8.46	75	1.68	20.0	1.26	13.40	15	0.52
7.0	5.6	100	10.22	85	1.82	17.8	1.25	15.17	20	0.60
8.5	6.0	100	12.78	112	2.13	16.6	1.41	18.67	25	0.67
4.5	4.2	150	7.20	48	1.70	35.7	1.07	11.43	10	0.45
6.3	5.0	150	10.00	80	2.00	30.0	1.26	16.00	15	0.52
7.0	5.6	150	12.40	105	1.85	26.8	1.25	18.75	20	0.60
8.5	6.0	150	13.80	125	2.30	25.0	1.41	20.83	25	0.67



**Fig. 3** Relationship between scour hole depth to tail water  $D_s/Y_t$  and  $Fr$  at apron solid



**Fig. 4** Relationship between scour hole length to tail water depth  $L_s/Y_t$  and  $Fr$  at apron solid



**Fig. 5** 3D model of bed channel downstream sharp-crested weir at apron solid 50-cm length, 25-cm weir height, and discharge 25 l/s

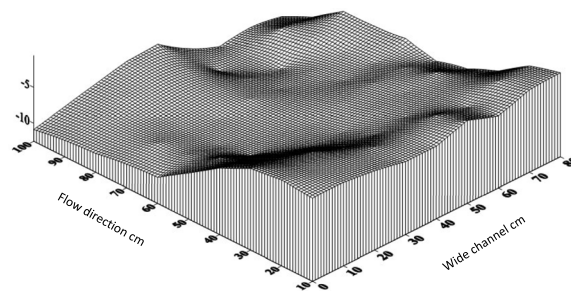
falls above the edge of the weir represents an area of disturbance and dispersal of the flow before its arrival. In the study area, when using a 50-cm-long protection floor, and due to the short distance, the flow is more turbulent when it reaches the sand layer than when the protection floor is 100 cm long due to the effect of the distance in reducing turbulence and calming flow [3].

#### Changing the scour geometries along the study area

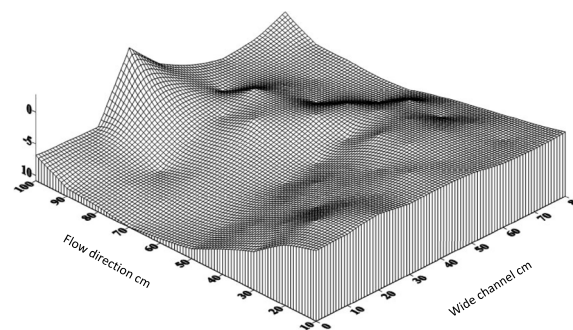
The change in the scour geometries of the study area is one of the essential points in study scour and sedimentation downstream the weir to show the effect of using different lengths of the apron solid in the form of the scour and its extension downstream the weir, through laboratory measurements of the topography of the sedimentary bottom of the study area after the end of the experiment run time and the stability of scour and sedimentation, which is variable longitudinal and transverse [15]. A 3D model of the sedimentary bed was drawn using the Surfer-13 program to show the change in the shape of scour and sedimentation that occurred downstream the weir when the length of the apron solid was increased and used maximum discharges (Figs. 5, 6 and 7).

Figure 5 represents a 3D model of the bed channel downstream sharp-crested weir of the experiments at apron solid with 50-cm length, 25-cm weir height, and discharge 25 l/s. This figure showed that the maximum scour occurred downstream the weir when using an apron solid of length 50 cm. This is due to the turbulence occurring downstream the weir due to the short distance of the flow of water, which moves toward the study area in the form of vortices of water that work with the force of its movement to transfer sand particles to a distant place.

Figure 6 represents a 3D model of a bed channel downstream sharp-crested weir of the experiments at apron solid with 100-cm length and 25 cm, weir height discharge of 25 l/s. This figure shows that when the length of the apron solid is increased to 100 cm, the depth and length of scouring decrease compared to the apron solid 50 cm because increasing the apron solid length reduces the energy of the flow that falls from above the crest of the weir before it reaches the study area as a result of the increase in the length of the distance through which the flow passes [6].



**Fig. 6** 3D model of bed channel downstream sharp-crested weir at apron solid 100-cm length, 25-cm weir height, and discharge 25 l/s



**Fig. 7** 3D model of bed channel downstream sharp-crested weir at apron solid 150-cm length, 25-cm weir height, and discharge 25 l/s

Figure 7 represents the 3D model of the bed channel downstream sharp-crested weir of the experiments at apron solid with 150-cm length, 25-cm weir height, and discharge 25 l/s. This figure shows that when using an apron solid of length 150 cm, the dimensions of scour hole increase compared to an apron solid with a length of 100 cm because of vortices forming downstream weir, and these vortices increased when the solid apron increased. This vortex caused increase in scour dimensions and then increased the depth and length of scour hole.

#### Relationship for calculating the maximum depth and length of scour concerning tailwater downstream the weir ( $D_s/Y_t$ , $L_s/Y_t$ )

It was reached to find two relationships to calculate the maximum depth and length of the relative scour hole  $D_s/Y_t$ ,  $L_s/Y_t$  by entering the data values, including the ratio between the depth and length of the scour hole to tailwater downstream the weir  $D_s/Y_t$ ,  $L_s/Y_t$ , and the values of the nondimensional variables which were obtained by applying the values of laboratory measurements in Eq. (3) which is  $\frac{D_s L_s}{Y_t} = f\left(Fr, \frac{Lb}{Y_t}, \frac{H}{Y_t}\right)$ . In the statistical program SPSS-21, which was arriving at a positional equation in terms of the approved variables, Eq. (5) is as follows:



$$\frac{Ds}{Yt} = 3.842 \cdot \frac{\left(\frac{H}{Yt}\right)^{0.601}}{(Fr)^{0.1} \left(\frac{Lb}{Yt}\right)^{0.282}} \tag{5}$$

After several attempts for other formulas similar to Eq. (5), the statistical analysis results proved an excellent effect of the variables adopted in this equation on the value of the coefficient of determination by ( $R^2 = 0.892$ ). The equation shows that the two expressions (5) ( $Fr$ ) and  $H/Yt$  have a good effect on the dependent variable  $Ds/Yt$ .

It was reached to find the weir position in the statistical program SPSS Statistics 21. A positional equation in terms of variables was approximated, which is Eq. (6).

$$\frac{Ls}{Yt} = 4.73 \cdot \frac{(Fr)^{0.749} \left(\frac{H}{Yt}\right)^{0.561}}{\left(\frac{Lb}{Yt}\right)^{0.25}} \tag{6}$$

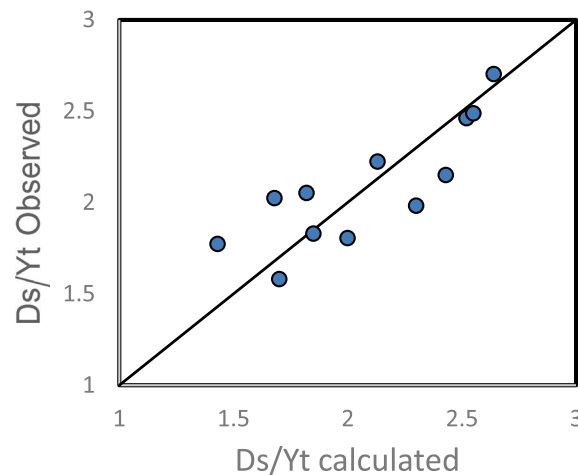
The statistical analysis results also showed a good effect of the dependent variables in Eq. (6) on the value of the coefficient of determination by  $R^2 = 0.804$ .

From the observation of the two equations (Eqs. 5 and 6), the ratio of the depth of the water above the edge of the weir to the depth of the tailwater  $H/Yt$  has a strong effect compared to the rest of the variables, which is more than it is in Eq. (5). Also, the impact of Froude number  $Fr$  is strong in Eq. (6) than it is in Eq. (5).

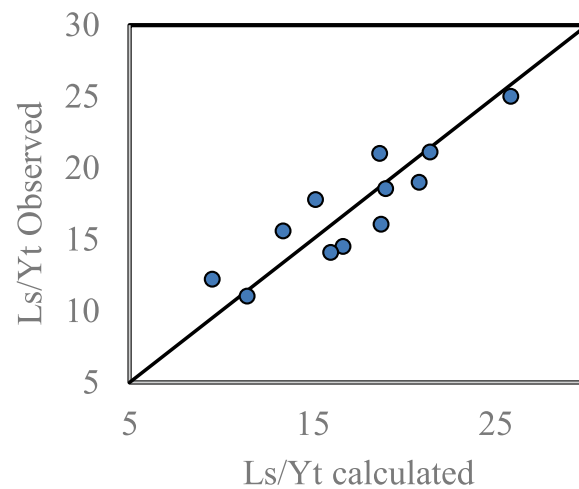
Figures 8 and 9 represent the comparison between experiments data and that calculated using Eqs. 5 and 6 of hole bore depth and length to tailwater ( $Ds/Yt$  and  $Ls/Yt$  observed and calculated respectively). From the figures, it can be seen a good agreement between observed and calculated data with a percentage error that does not exceed 10%.

### Conclusions

The scour was studied downstream sharp-crested, with a height of 25 cm, using three different distances of the solid apron 50, 100, and 150 cm. Uniform sand was used with a medium grain diameter of 0.5 mm and a standard deviation of  $\sigma$  equal to 1.3. It was



**Fig. 8** Relationship between observed and calculated values of  $Ds/Yt$  from Eq. 5



**Fig. 9** Relationship between observed and calculated values of  $L_s/Y_t$  from Eq. 6

found that the apron solid with a length of 50 cm causes greater scour as a result of the shorter running distance and the occurrence of turbulence and eddies close to the study area that cause scour. The depth and length of scour hole decrease when using an apron solid with a length of 100 cm in proportion to 19.3% and 27.7%, respectively, as a result of the increase in the distance that works to move away the turbulence of the water falling above the edge of the weir. The depth and length of scour hole increase when using an apron solid with a length of 150 cm by 7.4% and 10.4% due to vortices forming downstream weir, and these vortices increased when the solid apron increases. This vortex caused increase in scour dimensions and then an increased length of scour hole.

The statistical program SPSS 21 was used to arrive at two positional equations for the maximum depth and length of the scour hole in terms of variables and with a coefficient of determination of  $R^2 = 0.892$  and  $R^2 = 0.804$ , respectively.

#### Abbreviations

$D_s$	Peak height of the scour hole cm
$L_s$	Peak distance of the scour hole cm
$Q$	Discharge $L^3/T$
$H$	Head of water above the sharp-crested weir L
$W$	Width of the channel L
$L_b$	Length of apron solid L
$Y_t$	Tailwater depth L
$g$	Gravitational acceleration $L/T^2$
$P$	Weir depth L
$D_{50}$	Medium grain size diameter of the soil L
$\mu$	Dynamic viscosity of water $M/L.T$
$\rho$	Density of water $M/L^3$
$\rho_s$	Density of the sediment $M/L^3$
$Fr$	Froude number
$V$	Velocity of flow $L/T$
$\sigma$	Standard deviation
$R^2$	Coefficient of determination

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#### Authors' contributions

AM designed and constructed the weir model, NI provided the procedure of the experiment, and AM contributed to the interpretation of experimental results; all authors were involved in the writing and revision of the paper; and the authors read and approved the final manuscript.

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**Availability of data and materials**

The datasets used and analyzed during the current study are available from the corresponding author on reasonable request.

**Declarations****Competing interests**

The authors declare that they have no competing interests.

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**References**

1. Hamidifar H, Omid MH, Nasrabadi M (2011) Scour downstream of a rough rigid apron. *World Appl Sci J* 14(8):1169–1178
2. Alwan HH, Kamonna HH, Hashim NA (2016) Evaluation of local scour development downstream an apron of different angles for an ogee spillway. *Kufa J Eng* 7(3):1–12
3. Kumar, B., & Ahmad, Z. (2020). Experimental study on scour downstream of a piano key weir with a nose. 8th Iahrs Ishs, Santiago, Chile, May 12th to 15th. <https://doi.org/10.14264/uql.2020-595>.
4. Elnikhely EA, Fathy I (2020) Prediction of scours downstream of triangular labyrinth weirs. *Alex Eng J* 59(2):1037–1047
5. Rumann, J., Pavuček, M., & Dušička, P. (2021). Model investigations of scouring at the Hričov weir using short-range photogrammetry. In: IOP Conference Series: Materials Science and Engineering (Vol. 1203, No. 2, p. 022101). IOP Publishing.
6. Malla Obaida, Arwa Abdul Razzaq Jamal, (2021). "Effect of hydraulic variables on the scour for piano key weir type C". M.Sc. thesis, Department of Dams and Water Resources Engineering, College of Engineering, University of Mosul.
7. Lantz W, Crookston BM, Palermo M (2021) Apron and cut-off wall scour protection for piano key weirs. *Water* 13(17):2332
8. Kumar B, Ahmad Z (2022) Scour downstream of a piano key weir with and without a solid apron. *J Irrig Drain Eng* 148(1):04021066
9. Knauss, J. (2017). Prediction of critical submergence. In: Swirling "hydraulic apron solid" problems at intakes. Routledge, pp. 57–76.
10. R. Ghufran. (2011) Laboratory study of scour downstream screen walls used as energy dissipaters, M.Sc. thesis, Department of Dams and Water Resources Engineering. College of Engineering, University of Mosul.
11. British Standard Institution (1965), "Method of measurement of liquid flow in open channel". BS3680, part (A), London, England.
12. Dey S, Sarkar A (2006) Scour downstream of an apron due to submerged horizontal jets. *J Hydraul Eng* 132(3):246–257
13. Khattab N, Amin AMA (2015) Physical model study for mitigating local scour downstream of clear over-fall weirs. *Ain Shams Eng J* 6(4):1143–1150
14. Mohammed MY, Al-taee AY, Al-Talib AN (2011) Gravel roughness and channel slope effects on rectangular free overfall. *Damascus Univ J* 17:1
15. Dodaro G, Tafarojnoruz A, Sciortino G, Adduce C, Calomino F, Gaudio R (2016) Modified Einstein sediment transport method to simulate the local scour evolution downstream of a rigid bed. *J Hydraul Eng* 142(11):04016041
16. Ferraro D, Tafarojnoruz A, Gaudio R, Cardoso AH (2013) Effects of pile cap thickness on the maximum scour depth at a complex pier. *J Hydraul Eng* 139(5):482–491
17. Sharafati A, Tafarojnoruz A, Yaseen ZM (2020) New stochastic modeling strategy on the prediction enhancement of pier scour depth in cohesive bed materials. *J Hydroinf* 22(3):457–472