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Elastic buckling of simply supported stiffened steel plates with circular opening

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Abstract

Steel plates are used as the main components of steel elements in many structures. Openings in steel elements may exist for maintenance or inspection purposes in addition to creating paths for different fixtures such as air conditioning, pipes, and facilities. The structural performance of plates with openings is different from that of solid plates when subjected to axial compressive load. Openings affect local buckling, lateral torsional buckling, and shear of plates. Adding stiffeners around openings is a common engineering practice to strengthen plates. However, different design codes and guides provide estimates of the flexural and shear capacity of beams with no clear formulas for the influence of such openings on axial buckling strength. In this study, a parametric study has been performed using general-purpose finite element software, ANSYS, in order to study the effect of using stiffeners around circular opening in simply supported axially loaded plates on the elastic buckling behavior. Different parameters were considered including the aspect ratio of the plate, opening diameter, opening position in both horizontal and vertical directions, and stiffener's dimensions. The results of the extensive parametric study are used to propose an empirical design formula considering the different studied parameters.

Keywords: Elastic buckling, Finite element modeling, Stiffeners, Web opening

Introduction

Introducing openings to plates affects their structural behavior. Instability/buckling can occur under axial compressive loads at loading levels lower than anticipated. Common design practices usually include strengthening of perforated plates with stiffeners around openings. The AISC design guide [1] provides design formulas to estimate the shear and flexural capacity of steel beams with web openings. However, the guide focuses on the case of plates subjected to linear stress gradient without accounting for stiffener dimensions and opening location. Moreover, while the formulae proposed by the guideline are simple and useful to apply to real situation, it needs more research to account for different parameters and factors [2]. Elastic buckling and elasto-plastic buckling of perforated plates and I-beams have been intensively investigated by several researchers through experimental and numerical analyses [3–17]. Performed research focused on the influence of different opening shapes, size of opening, and location of opening in addition to plate boundary conditions. Results from numerical and

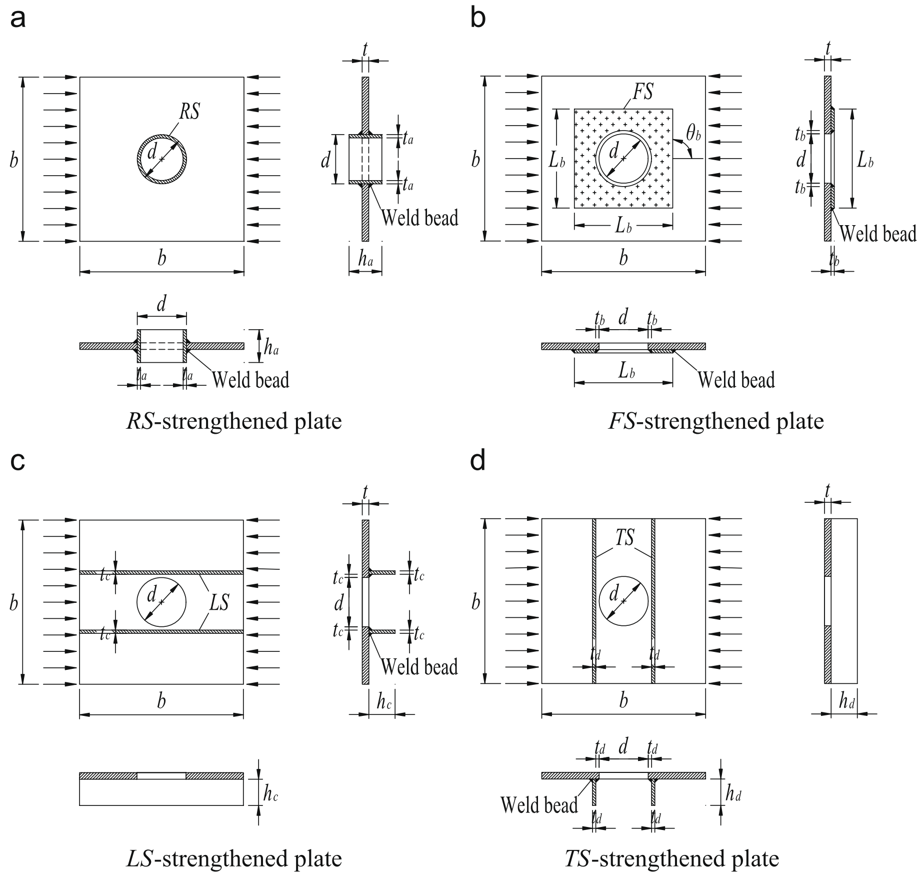


Fig. 1 The stiffened square plate with a central circular hole (Cheng and Zhao (2010) [24])

experimental investigations were used to build interaction diagrams and design curves [3, 5, 7, 8, 18–21] or proposing design formulas that account for different parameters [15, 20, 22]. In addition, recommendations regarding critical zones in perforated plates where it is not recommended to place openings and value of critical buckling strength were evaluated by several researchers [8, 13, 23]. Cheng and Zhao [24] were the first to study the strengthening of simply supported square steel plates with a centrally placed circular hole subjected to uniaxial compressive loads using stiffeners. Four types of stiffeners were considered including ringed stiffener, flat stiffener, longitudinal stiffener, and transverse stiffener as shown in Fig. 1. A series of numerical analyses of both elastic and elasto-plastic buckling behaviors for stiffened and unstiffened perforated plates were carried out. However, the study focused on strengthened plates with central circular openings. In addition, stiffener plates were extended through the whole width or depth of perforated plates which might not be practical in some cases. In the current research, an extensive parametric study is performed to study the effect of strengthening on elastic buckling of simply supported plates while considering different locations for openings. Parameters also include plate's aspect ratio (α), width-to-thickness ratio (λ), circular opening diameter, and opening location with respect to X and Y axes. Results are analyzed to pinpoint the key parameters. Accordingly, a

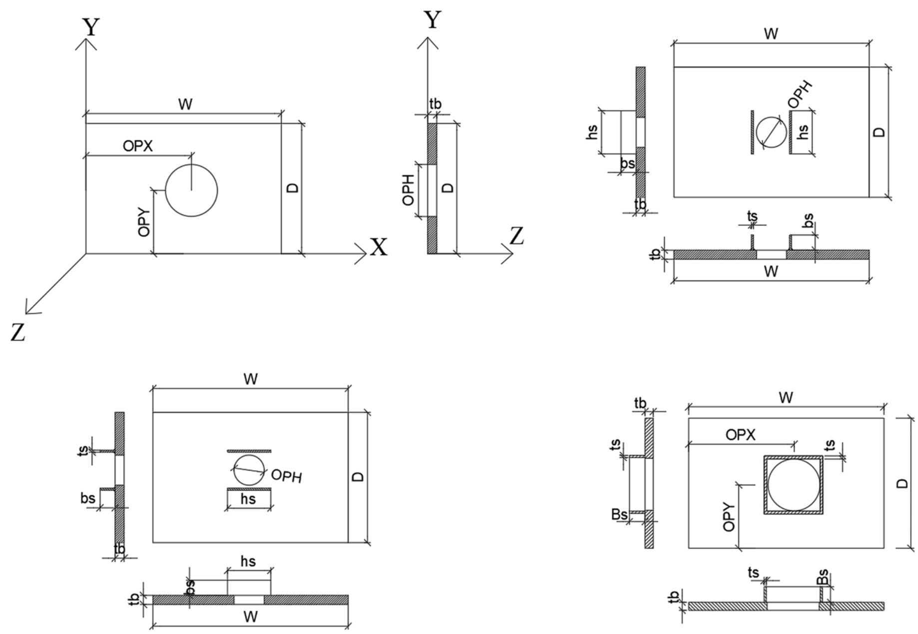


Fig. 2 Geometry of models and stiffener configurations

formula is developed to estimate the elastic buckling coefficient of unstiffened and stiffened plates with circular openings.

Methods

Description of models

SHELL63 element is used to model plates and stiffeners using ANSYS 11 finite element software [25]. This element has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal *x*, *y*, and *z* directions and rotations about the nodal *x*, *y*, and *z* axes.

Models in the current study are divided into two main groups: (1) plates with circular unstiffened opening and (2) plates with circular stiffened opening. Stiffening configurations included stiffeners parallel to load direction, stiffeners perpendicular to load direction, and stiffeners forming box shape around the opening as illustrated in Fig. 2. Modeled plates have length *W*, height *D*, and thickness t_b. Circular openings have diameter OPH, edge distance from the *Y*-axis OPX, and edge distance from the *X*-axis OPY. In addition, stiffener dimensions are length H_s, breadth B_s, and thickness t_s.

Boundary conditions have a large influence on the buckling resistance of plates as they control behavior during buckling. Web plates are usually considered to be simply supported or partially fixed. For the top end of the model, the top area of the steel plate was constrained in *X* and *Z* directions to represent hinged support at the top end of the plate. Several meshing trials were made for solid plate and compared to the theoretical equation proposed by Timoshenko [26] till the most fitting mesh was reached. Figure 3a shows boundary conditions considered for the different models in

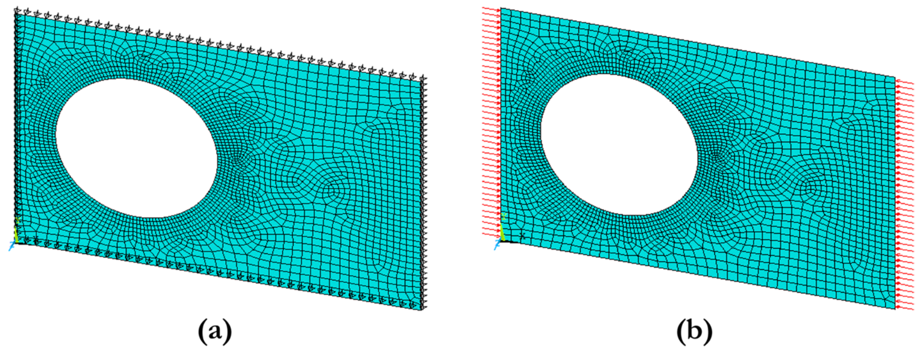


Fig. 3 Boundary conditions and applied stresses for the example model

Table 1 Geometry and mechanical properties of verification models (solid plates)

	W/D		
	0.5	**1**	**1.4**
***W*(mm)**	1000	1000	1400
***D*(mm)**	2000	1000	1000
Buckling factor (K_{no})	6.25	4	4.47
$б_{cr}$(Timoshenko [26])	29.66	75.92	84.84
$б_{cr}$(ANSYS model)	29.631	75.876	84.789
Error	0.098%	0.058%	0.060%

the current study. Models are analyzed under uniaxial compressive stresses. Pressure is applied at two opposite edges as shown in Fig. 3b.

Eigenvalue buckling analysis

Buckling load and subsequently buckling coefficient (K) are estimated through eigenvalue buckling analysis for the unloaded steel plates. The buckling coefficient is determined considering the following formula:

$$K = \frac{12\sigma_{cr}\left(1-\nu^2\right)}{\pi^2 E}.\left(\frac{D}{t}\right)^2 \quad (1)$$

Where

$б_{cr}$: Critical buckling stress calculated as per theoretical or numerical methods

ν: Poisson's ratio

E: Young's modulus

D and t: Plate dimensions as defined in Fig. 2

The buckling factor (K) was given a suffix to indicate whether it is calculated for the solid plate (K_{no}), plate with circular opening (K_o), or strengthened plate with circular opening (K_s).

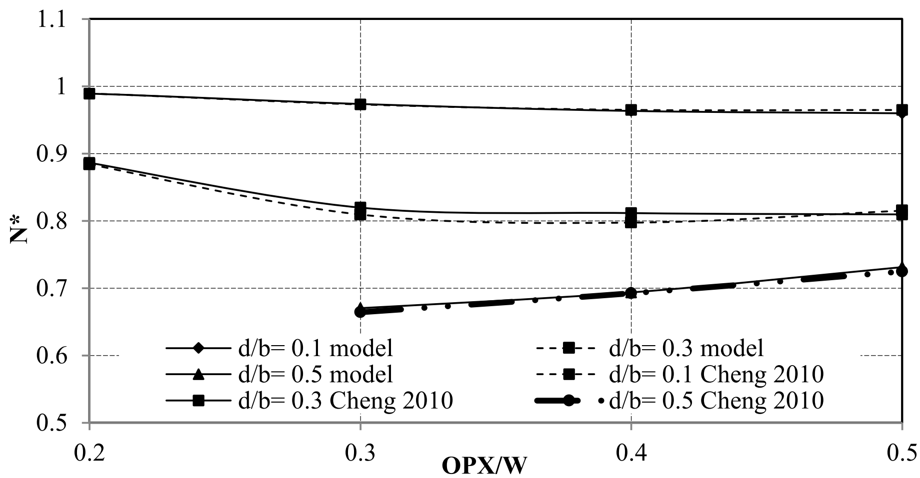


Fig. 4 Verification of un-strengthened plates with openings

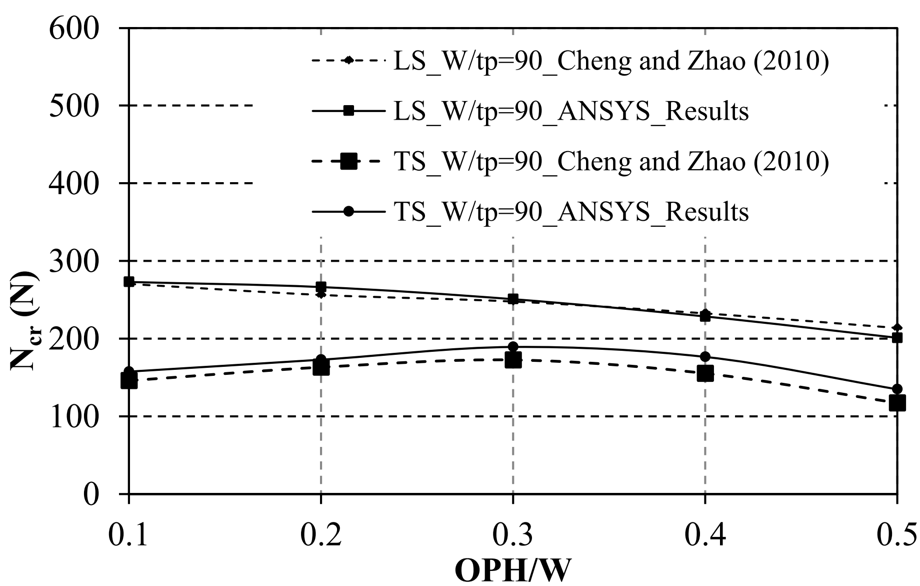


Fig. 5 Verification of strengthened plates with openings strengthened with longitudinal and transversal stiffeners

Model verification

Several initial runs are performed to test the sensitivity of the used mesh and determine the best size that will guarantee suitable run time and accuracy. Afterwards, verification of models is performed first for solid plates against elastic buckling values estimated by Timoshenko [26] as listed in Table 1.

Models for plates with circular openings are built and verified against results reported by Cheng and Zhao [24] for elastic buckling of a rectangular plate with circular opening under constant load. Figure 4 shows the effect of opening position and opening diameter on the elastic buckling load resulting in the current study compared to values reported by Cheng and Zhao [24]. Good agreement is observed between the

Table 2 Parametric study independent variables

	Parameter	Range of study
Plate properties	$\alpha = W/D$	0.5, 1, 1.5, and 2
	W/t_p	100, 125, 200
Opening dimensions	H=OPH/W	0.1–0.5
	X=OPX/W	0.1–0.5
	Y = OPY/D	0.1–0.5
Stiffener dimensions	B_s/t_s	10
	β=B_s/OPH	0.4–0.8
	H_s/OPH	1.1, 1.3, and 1.5

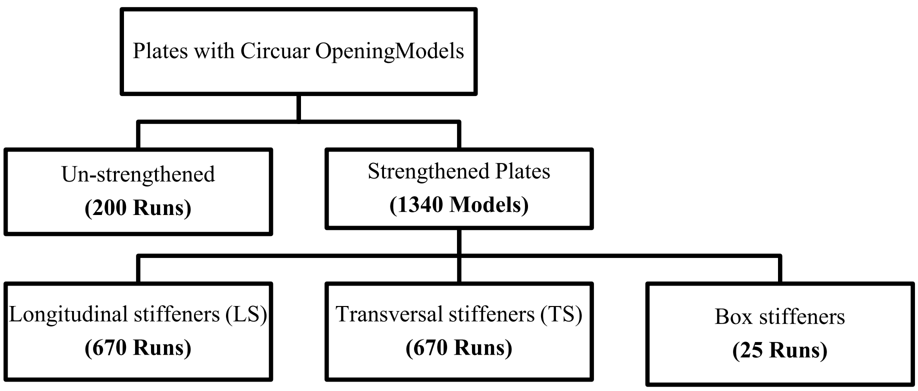


Fig. 6 Performed analyses in the current study

results of the mentioned paper compared to the results extracted from the analytical model. Figure 4 shows that the maximum error between results reaches 2% which is considered a small value and can be accepted.

Figure 5 shows the verification of strengthened plates against results reported by Cheng and Zhao [24] for longitudinal and transversal stiffeners. Results show good agreement with a maximum error of 10%. The difference is due to neglecting transverse shear deformations in the current study.

Independent parameters

The parametric study is performed using the calibrated analytical models to investigate the effect of different parameters that may affect the elastic buckling behavior of un-strengthened and strengthened plates with circular opening under uniaxial compressive loading. The studied independent parameters used in the study can be summarized as shown in Table 2. The considered independent parameters include the width-to-depth ratio of the plate (W/D), width-to-thickness ratio of the plate (W/t_p), location of the opening in X- and Y-directions, and dimensions of the added stiffeners (H_s, B_s, and t_s). The parametric study is established by varying the value of each independent parameter separately. Figure 6 summarizes performed analyses and symbols used throughout the performed study.

Several runs are also performed considering stiffeners forming a box shape around the opening. This trial was made to check whether combining the stiffeners parallel to the load direction with those normal to the load direction will enhance elastic buckling behavior or not. The additional runs are executed only considering $\alpha = 1.0$ combined with stiffeners' dimensions with respect to the circular opening diameter (B_s/OPH $=$ 0.6) for opening moving in *X*- and *Y*-directions.

Results and discussion

An extensive parametric study is conducted to evaluate the effect of the different geometric parameters on the elastic buckling of strengthened and un-strengthened plates with circular openings in order to deduce a suitable formula for the buckling coefficient (*K*). Effects of the different considered parameters will be exhibited in the next sections for un-strengthened and strengthened plates separately.

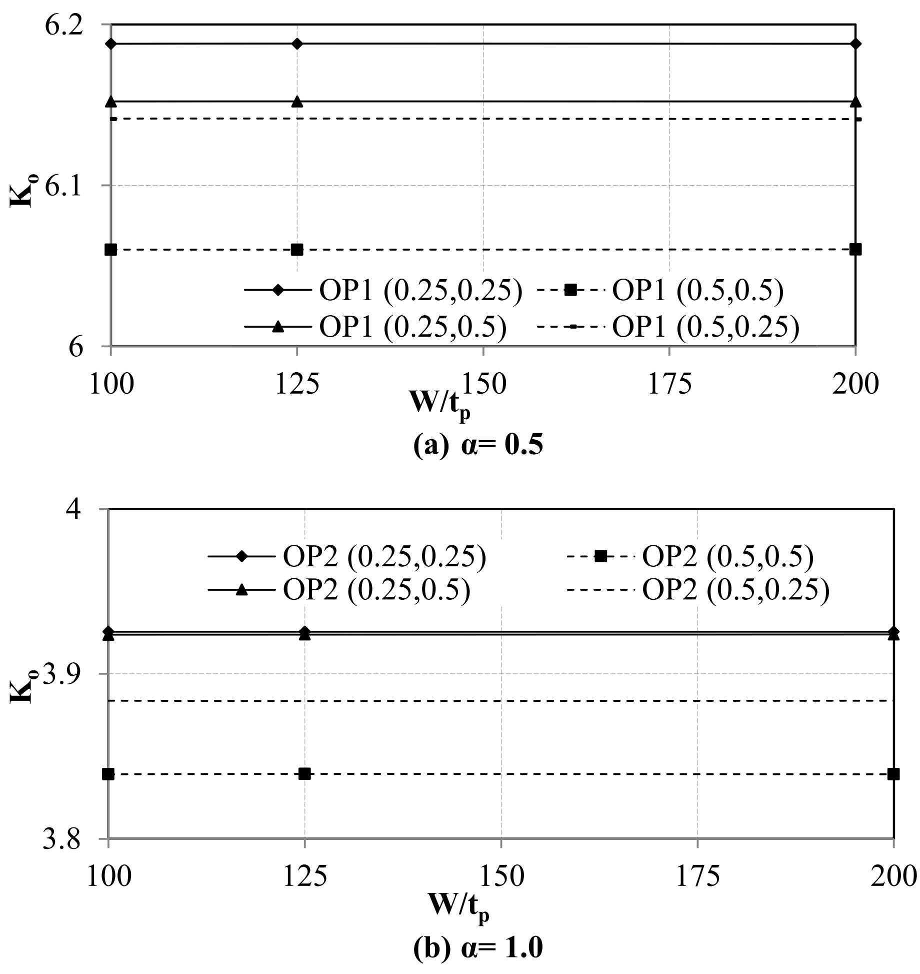


Fig. 7 Variation of buckling coefficient (K_o) vs. plate slenderness ratio

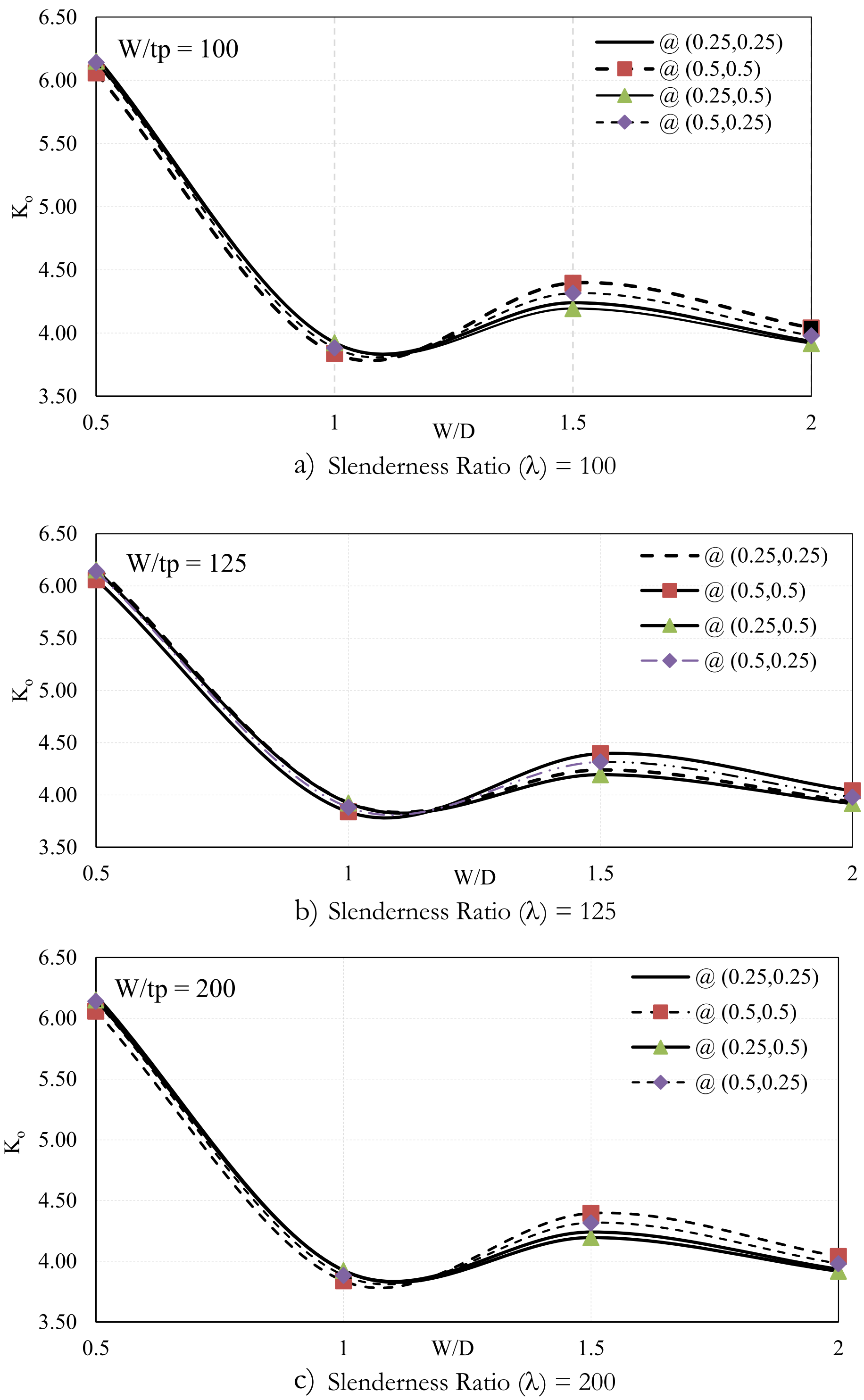


Fig. 8 Variation of buckling coefficient (K_o) vs. plate aspect ratio. **a** Slenderness ratio (λ) = 100. **b** Slenderness ratio (λ) = 125. **c** Slenderness ratio (λ) = 200

Un-strengthened plate with circular opening

Influence of slenderness ratio (W/t_p)

Several runs were performed to study the influence of plate slenderness ratio on the buckling coefficient factor (K) as illustrated in Fig. 8, where symbols OP1, OP2, and OP3 present opening diameter-to-width ratios of 0.1, 0.2, and 0.3, respectively. Numbers between brackets refer to the position of the opening center with respect to the X and Y axes. Figure 7 shows that the plate slenderness ratio has no effect on the buckling factor coefficient (K_o). If the buckling coefficient (K_o) for perforated plates is compared to the ones extracted from solid plate models, it can be observed that the coefficient (K_o) decreases by about 1–2.5% for plates with an opening near the corner. However, for models having small diameters, the decrease in buckling coefficient can be neglected. For plates having an aspect ratio equal to 1.5 and a central opening, the buckling coefficient of perforated plates slightly increased when compared to solid plates.

Influence of aspect ratio (W/D)

As indicated by previous researchers, the aspect ratio of plates has a great effect on the buckling coefficient factor (K). Figure 8 shows that the buckling coefficient factor (K_o) decreases linearly when the aspect ratio is less than 1 with a high slope. However, for aspect ratios ranging between 1 and 1.5, the buckling coefficient factor (K_o) increases slightly then decreases again as the aspect ratio reaches to 2. It is also clear from curves that the general relationship between the buckling coefficient factor (K_o) and plate aspect ratio is the same for all considered slenderness ratios.

Influence of opening position in the X-direction (OPX/W)

For a small aspect ratio equal to 0.5 and a small opening diameter, the position of the hole affects the buckling coefficient factor significantly. As shown in Fig. 9, the local buckling factor (K) decreases as the opening moves towards the center of the plate. While for aspect ratios equal to 1 till 2, the buckling coefficient factor (K_o) slightly changes as the position of open changes.

For the different considered aspects, the buckling coefficient factor (K_o) decreases as the opening gets near to the center. The decrease ranges from 4 to 10.4% for the aspect

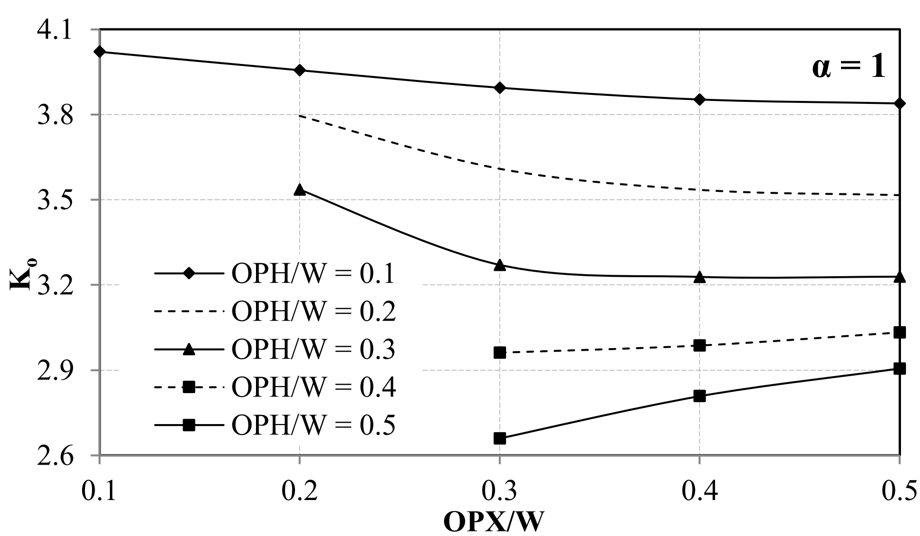


Fig. 9 Variation of K_o vs. opening position in the X-direction for $\alpha = 1$ and different opening diameters

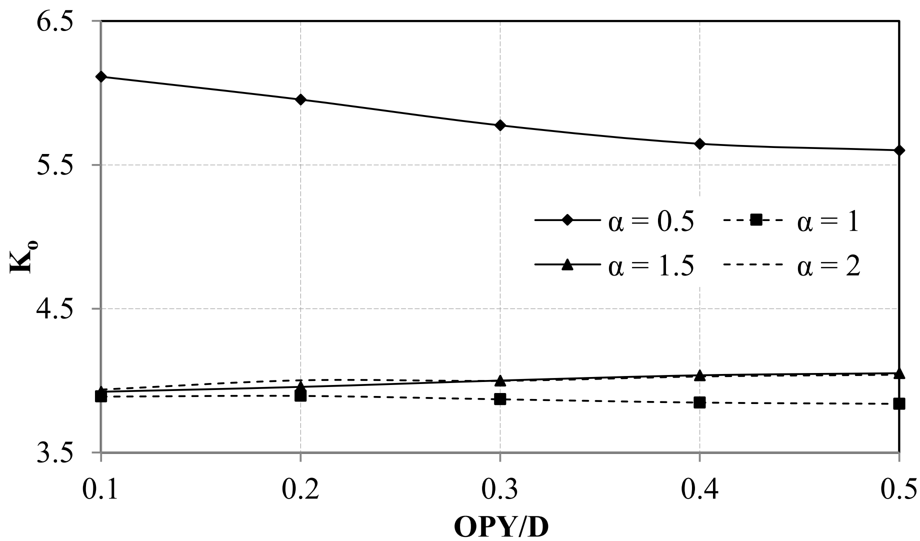


Fig. 10 Variation of K_o vs. opening position in the *Y*-direction for different aspect ratios

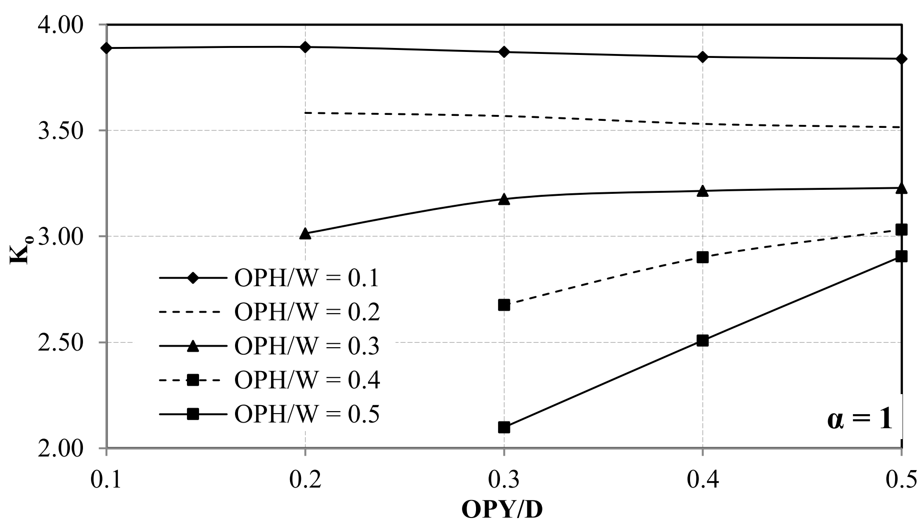


Fig. 11 Variation of K_o vs. opening position in the *Y*-direction for $\alpha = 1$ and different opening diameters

ratio equal to 0.5. Meanwhile, the decrease in buckling coefficient factor (K_o) ranges from 2.8 to 4.1% for the aspect ratio equal to 1.0. For the aspect ratio equal to 2, the buckling coefficient factor (K_o) decreases as the opening gets near to the center except for the case of openings near the plate corner. In this case, a slight increase may occur and reaches 0.5%. While the decrease ranges from 1 to 4%. This shows that for aspect ratios larger than 1.0, the effect of the opening is less pronounced than other aspect ratios.

Influence of opening position in the Y-direction (OPY/W)

The position of the opening in the *Y*-direction is varied for different plate aspect ratios and the influence on the local buckling factor (K_o) is studied. It is generally observed that the buckling factor decreases as the opening approaches the center of the plate.

Meanwhile, for an aspect ratio equal to 1 through 2, the buckling coefficient factor (K_o) slightly changes as the position of the open changes as shown in Fig. 10.

Figure 11 shows the effect of the opening diameter on the buckling coefficient factor (K_o) at the same aspect ratio which was taken to be 1. The results show that the buckling coefficient factors (K) decrease as the opening diameter increases.

Strengthened plate with longitudinal stiffener

Several runs are performed while considering longitudinal stiffeners around the introduced circular opening. Two cases are considered for the location of the circular opening: (1) a circular opening moving in the X-direction and centered in the Y-direction and (2) a circular opening moving in the Y-direction and centered in the X-direction. These cases are considered for circular openings having OPH/W ratios ranging between 0.1 and 0.5 while considering plate aspect ratios of 0.5, 1, 1.5, and 2. Accordingly, the buckling coefficient factor for stiffened plate (K_s) is compared to the buckling coefficient

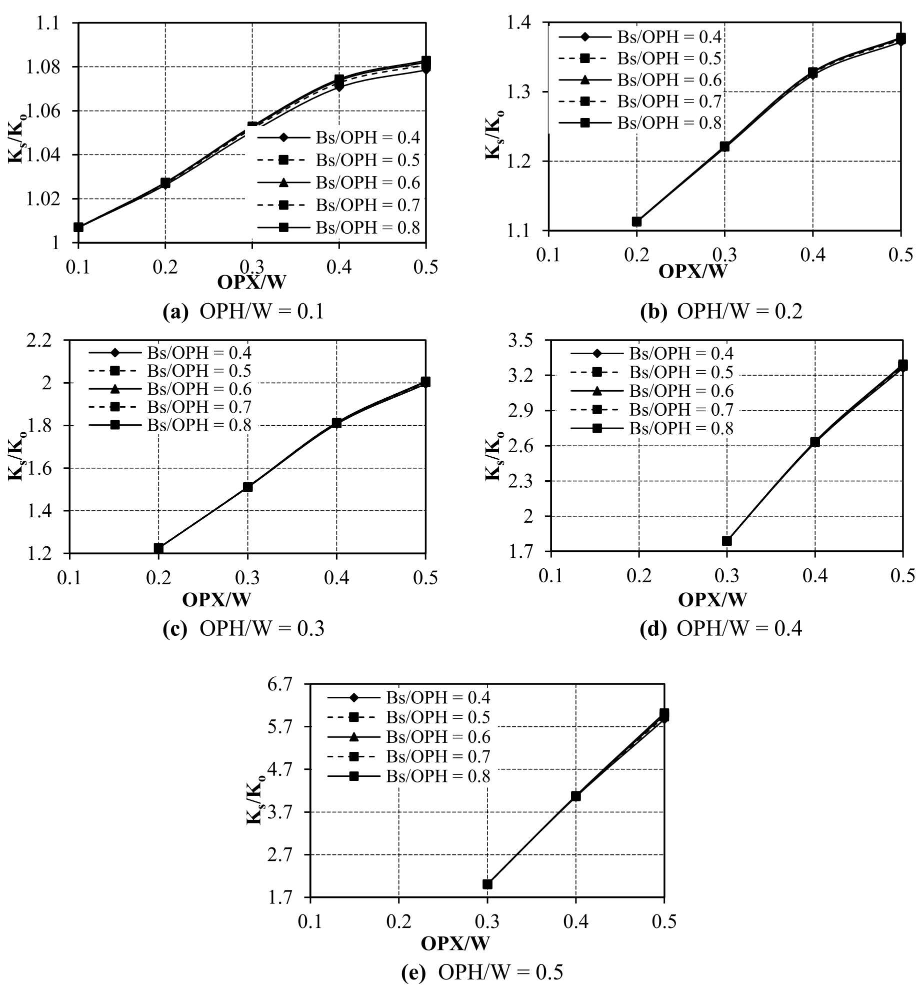


Fig. 12 Normalized buckling coefficient factor (K_s/K_o) vs. normalized opening position (B_s/OPH = 0.4–0.8). **a** OPH/W = 0.1. **b** OPH/W = 0.2. **c** OPH/W = 0.3. **d** OPH/W = 0.4. **e** OPH/W = 0.5

factor for unstiffened plates with a circular opening (K_o) and the buckling coefficient factor of the solid plate (K_{no}).

Results are exhibited for different plate aspect ratios while comparing normalized buckling factor with respect to the un-strengthened plate with opening (K_s/K_o) and normalized buckling factor with respect to the solid plate (K_s/K_{no}). Figure 12 shows the relation between the normalized buckling factor coefficient (K_s/K_o) with the position of the cutout with respect to the *y*-axis (OPX/W) and variation of B_s/OPH for plate aspect ratio equal to 0.5.

It can be deduced from the curves that:

- For OPH/W = 0.1, as the location of the opening in *X*-direction with respect to plate width (OPX/W) changes from 0.1 to 0.5, the buckling coefficient factor ratio (K_s/K_o) increases by a percentage ranging from 101 to 108.3%. Meanwhile, for OPH/W = 0.2, the buckling coefficient factor ratio (K_s/K_o) increases by a percentage ranging from 112.5 to 137.8%. This ratio increases between 122.5–200.7% and 178.9–329.6% for the OPH/W ratio equal to 0.3 and 0.4, respectively.
- The results also show that the change of stiffener breadth (B_s) has no effect on the buckling coefficient factor (K_s).

For aspect ratio 0.5, Fig. 13 shows the relation between the normalized buckling factor coefficient (K_s/K_o) with the variation of B_s/OPH and the position of the cutout with

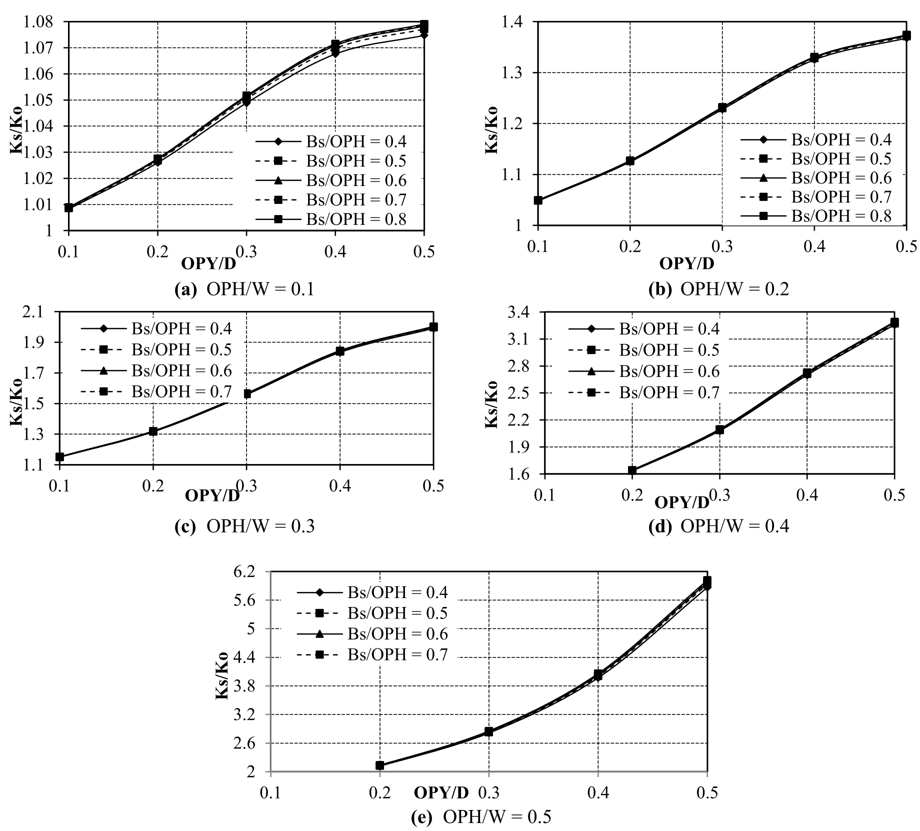


Fig. 13 Normalized buckling coefficient factor (K_s/K_o) vs. normalized opening position (B_s/OPH = 0.4–0.8). **a** OPH/W = 0.1. **b** OPH/W = 0.2. **c** OPH/W = 0.3. **d** OPH/W = 0.4. **e** OPH/W = 0.5

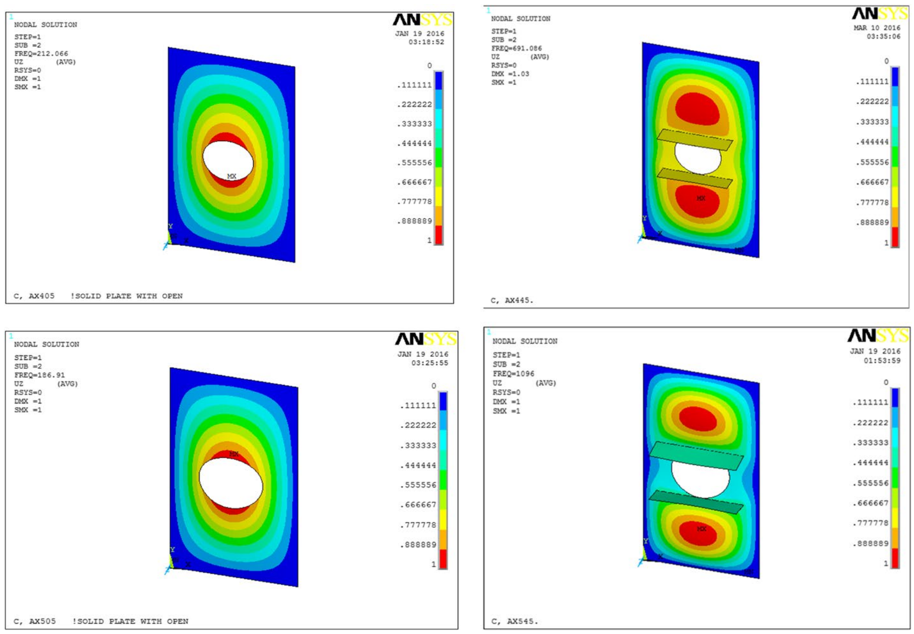


Fig. 14 Deformed shape for stiffened and unstiffened plates

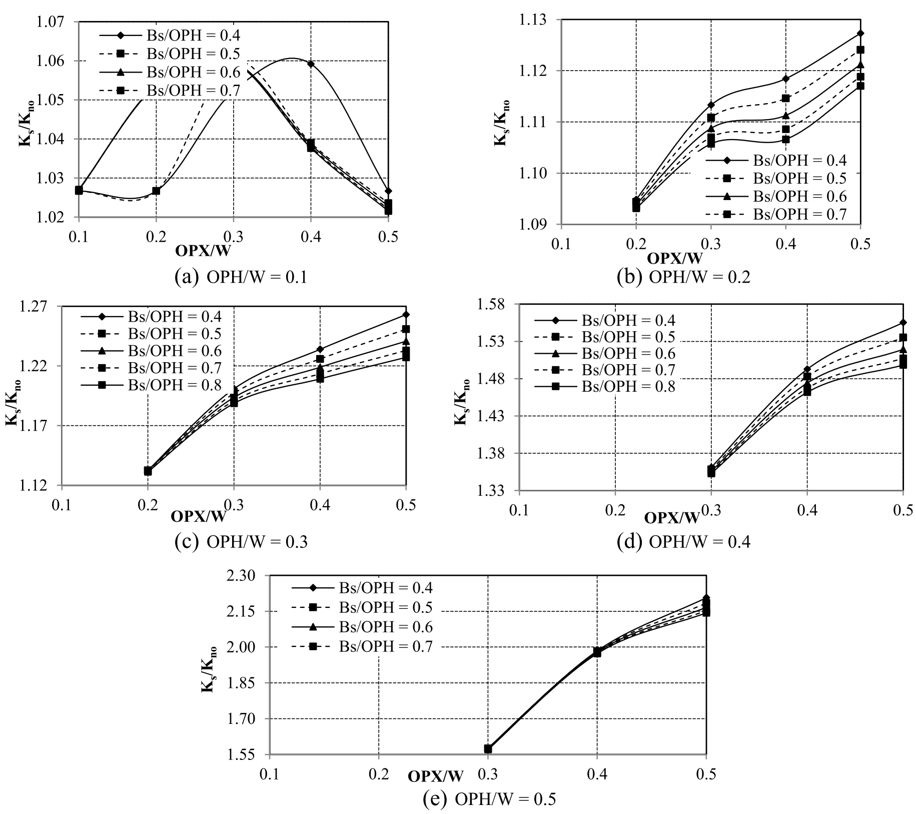


Fig. 15 Change of normalized buckling coefficient factor K_s/K_{no} with respect to normalized distance from the *Y*-axis OPX/W, (Bs/OPH = 0.4–0.8). **a** OPH/W = 0.1. **b** OPH/W = 0.2. **c** OPH/W = 0.3. **d** OPH/W = 0.4. **e** OPH/W = 0.5

respect to the x-axis (OPY/D). Examining the figure, it is clear from the curves that as the OPY/D ratio increases, the buckling coefficient factor ratio (K_s/K_o) increases. The percentage of increase ranges between 104.7 and 601.7%. The maximum increase is observed when OPH/W = 0.5 and OPY/D = 0.5.

Figure 14 shows the deformed shape for stiffened and unstiffened plates. It can be seen that the horizontal stiffener changes the elastic buckling mode with respect to the number of waves. This is due to the increase in plate stiffness upon strengthening.

Figure 15 shows a comparison between stiffened perforated plate and solid plate for an aspect ratio of 1.5. It is clear from the curves that the influence of adding stiffener

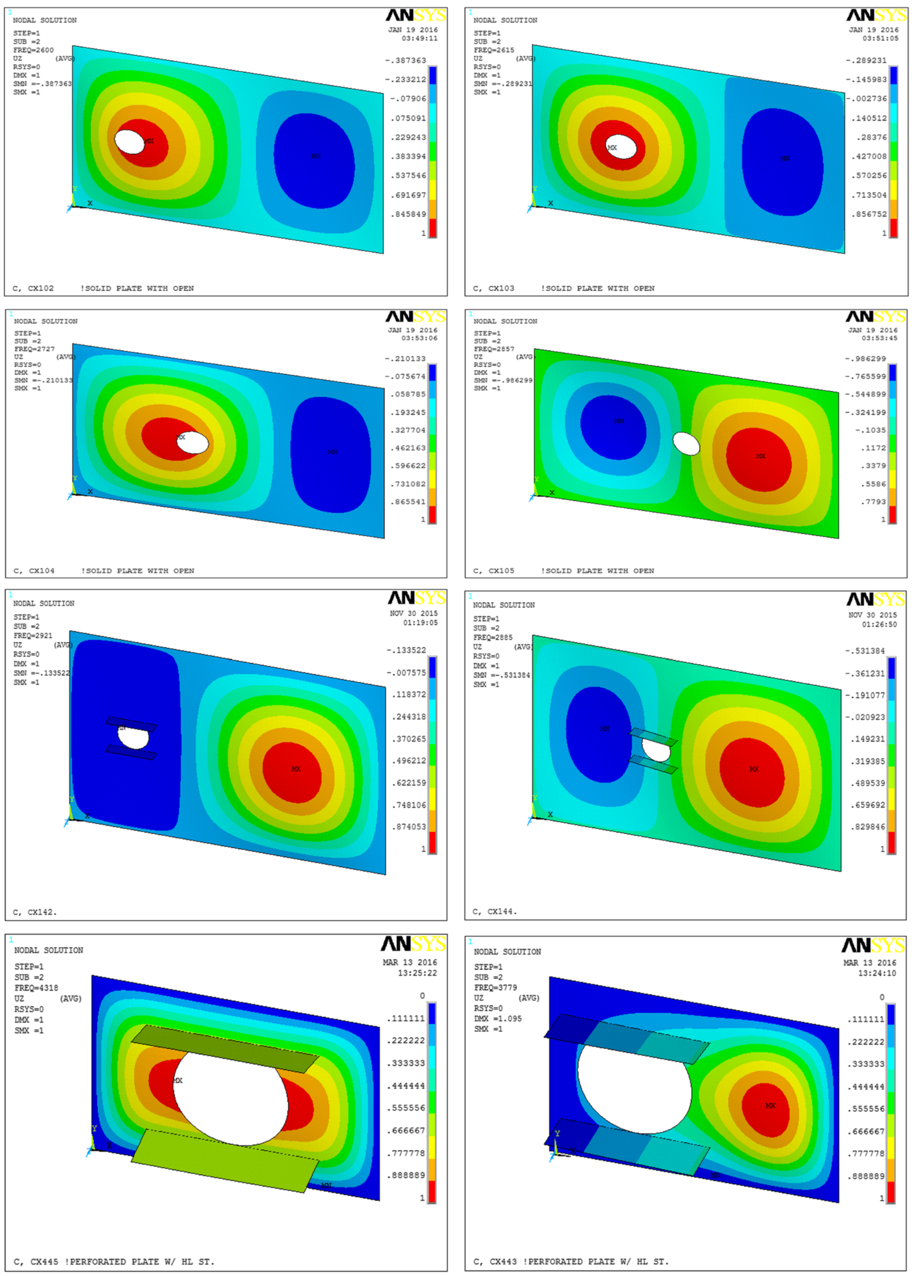
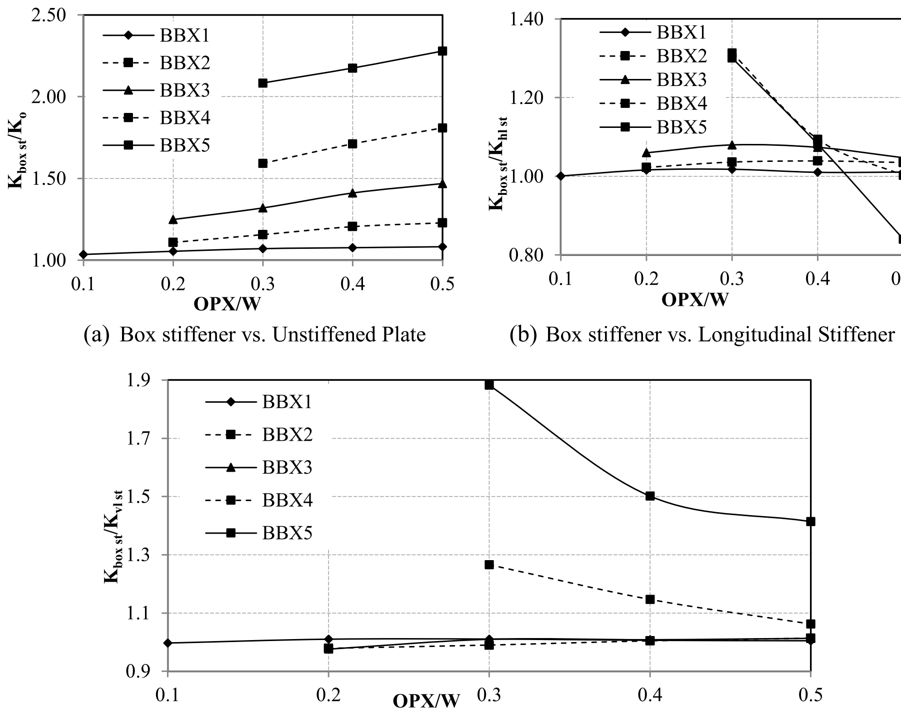


Fig. 16 Deformed shape for different considered plates



(a) Box stiffener vs. Unstiffened Plate

(b) Box stiffener vs. Longitudinal Stiffener

(c) Box stiffener vs. Transversal Stiffener

Fig. 17 Effect of the box stiffener considering circular opening moving in the *X*-direction. **a** Box stiffener vs. unstiffened plate. **b** Box stiffener vs. longitudinal stiffener. **c** Box stiffener vs. transversal stiffener

for OPH/W = 0.1 is different than other ratios. At this ratio, the local buckling factor increases at OPX/W = 0.1–0.3 then it starts to decrease at OPX/W = 0.4 and 0.5. For the other ratios, the buckling coefficient increases as OPX/W increases by a percentage ranging between 108 and 214.3%. In addition, it can be observed that the stiffener breadth (B_s) value does not have a large influence on the local buckling coefficient (K_s) for the different values of OPH/W. Figure 16 shows the deformed shape of stiffened and unstiffened plates as the opening changes its location in the *X*-direction. When the plate aspect ratio exceeds 1.0, the plate deforms in two half waves. Adding a circular opening near the edge or at the middle of these plates has relatively no effect on the plate deformed shape. For other locations of openings, it is observed that deformation of the part including the opening is larger than the one in the other half that does not include an opening. When adding stiffeners parallel to the load direction, a decrease in deformations is generally observed. Increasing the opening diameter gradually changes the deformed shape to a single wave which explains the decrease of the local buckling coefficient for stiffened plates.

Strengthened plate with transversal stiffener

Several runs are performed while considering transversal stiffeners around the introduced circular opening. Two cases are considered for the location of the circular opening: (1) a circular opening moving in the *X*-direction and centered in the *Y*-direction and (2) a circular opening moving in the *Y*-direction and centered in the *X*-direction. These cases are considered for circular openings having OPH/W ratios ranging between 0.1 and 0.5 while considering plate aspect ratios of 0.5, 1, 1.5, and 2. Accordingly, the buckling coefficient factor for stiffened plate (K_s) is compared to the buckling coefficient factor for unstiffened plates with a circular opening (K_o) and the buckling coefficient factor of the solid plate (K_{no}).

Strengthened plate using box stiffener

Combining longitudinal and transversal stiffeners, a box stiffener is introduced around the opening. Several runs were made on plates with an aspect ratio of 1.0 only to compare the effect of the box stiffener with each case separately. Figures 18 and 19 show results compared to unstiffened opening and opening stiffened with longitudinal and transversal stiffeners, respectively.

Examining Fig. 17, it is clear that the box stiffener strengthens the plate. When compared to longitudinal stiffener, using the box stiffener has a small effect for OPH/W = 0.1–0.3. However, its effect increases clearly for OPH/W = 0.4 and 0.5. For OPX/W = 0.3, strength increases by about 40%; meanwhile, it decreases by 20% when OPX/W = 0.5. This can be attributed to the fact that the box stiffener is totally inside the maximum deformed zone and has no effect on the deformed shape. When comparing box stiffener to transversal stiffener, the influence can be neglected when OPH/W = 0.1–0.3. However, strength increases by a percentage of 90% when OPH/W = 0.5 and OPX/W = 0.3. Figure 18 shows the comparison between the box stiffener and the transversal stiffener.

It can be seen that using the box stiffener strength doubles the elastic buckling strength when compared to a longitudinal stiffener for OPH/D = 0.5 and OPY/D = 0.3. However, for OPH/D = 0.1–0.3, the box stiffener strength provides the same strength

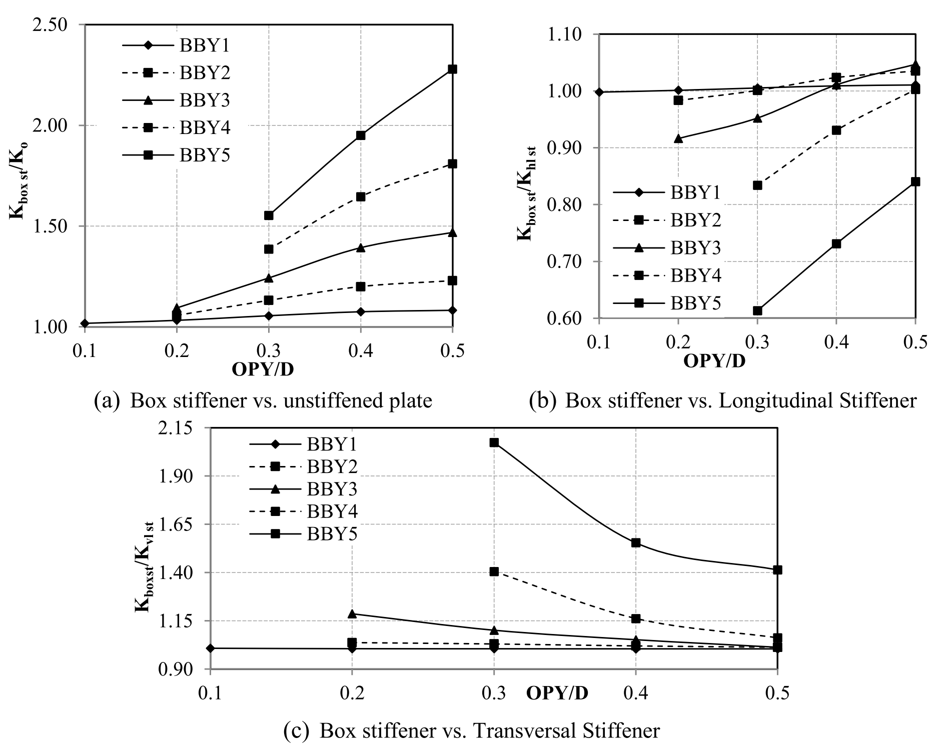


Fig. 18 Effect of the box stiffener considering circular opening moving in the *Y*-direction. **a** Box stiffener vs. unstiffened plate. **b** Box stiffener vs. longitudinal stiffener. **c** Box stiffener vs. transversal stiffener

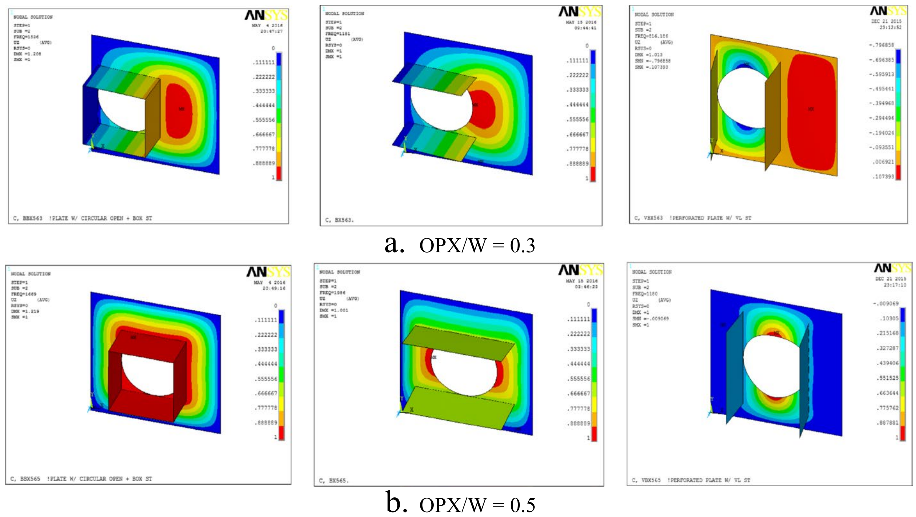


Fig. 19 Deformed shape for the box stiffener vs. longitudinal and transversal stiffeners for OPH/W = 0.5. **a** OPX/W = 0.3. **b** OPX/W = 0.5

of transversal stiffeners. Considering Figs. 19 and 20, it is clear how the deformed shape changes with the change of stiffener type and that explains the trend of curves above.

Proposed formula for elastic buckling of un-strengthened and strengthened plates with circular openings

Timoshenko [26] proposed a formula to calculate the buckling stress for plates with different aspect ratios, slenderness ratios, loading conditions, and boundary conditions as follows:

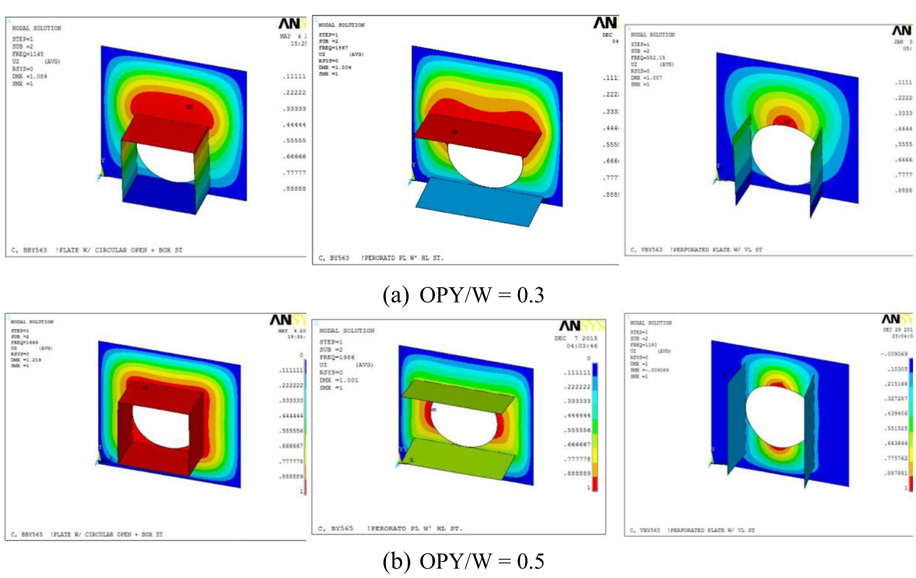
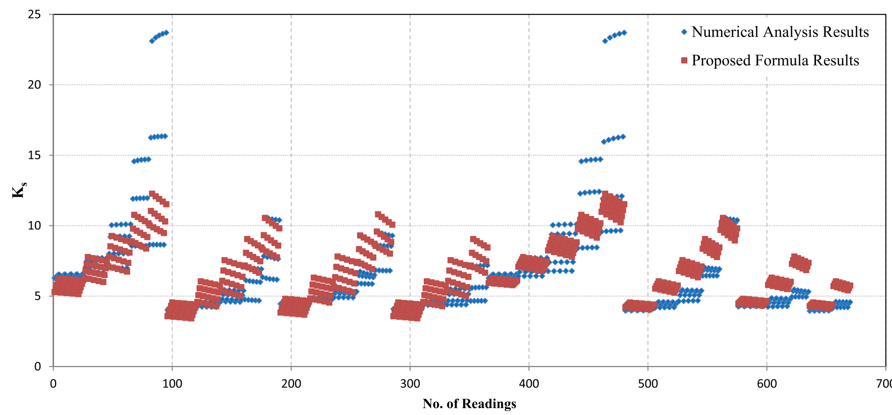
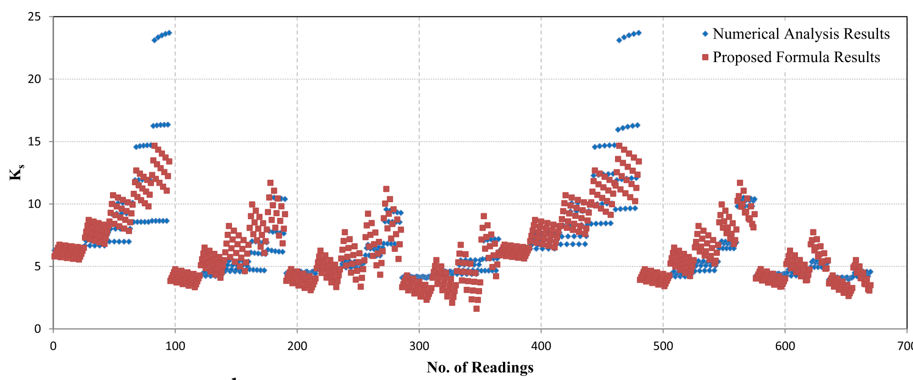


Fig. 20 Deformed shape for the box stiffener vs. longitudinal and transversal stiffeners for OPH/W = 0.5. **a** OPY/W = 0.3. **b** OPY/W = 0.5



a. Stiffeners parallel to load direction for plates with circular opening



b. Stiffeners perpendicular to load direction for plates with circular opening

Fig. 21 Verification of proposed formulas. **a** Stiffeners parallel to load direction for plates with a circular opening. **b** Stiffeners perpendicular to load direction for plates with a circular opening

$$\sigma_{cr} = \frac{(N_x)_{cr}}{t} = k\frac{\pi^2 E}{12(1-\upsilon^2)\left(^{D}/_{t}\right)^2} \quad (2)$$

The main purpose of the current study is to discuss the effect of different parameters on local buckling of plates and to propose an empirical formula to calculate the local buckling coefficient (K_o) for a rectangular plate with circular opening having different positions in X- and Y-directions. Based on the parametric study results, the plate slenderness ratio proved to have a neglected effect on the buckling coefficient. Hence, parameters considered in the proposed equation include 5 factors (aspect ratio α, slenderness ratio λ, normalized opening diameter OPH/W, normalized distance between the center of open to the Y-axis OPX/W, normalized distance between the center of the circle and the X-axis OPY/D). Several formulas were checked out through many trials using the Eureqa program [27]. The program was used to run a formula search using regression analysis after defining the detrimental factors. Estimated formulas with their different complexity factors and errors were calculated. The most suitable equation was then chosen according to the minimum calculated error. These are shown in Eqs. 3 to 5 for unstiffened and stiffened plates with circular openings. The accuracy of the proposed equations is compared in view of the parametric study results as shown in Fig. 21. The R^2 value was found to range between 0.86 and 0.98 which can be considered a good match.

$$\text{Unstiffened plate } \mathrm{K_o} = 0.97 * \mathrm{K} - 2.86 * \mathrm{X} * \mathrm{H} - 0.97 * \mathrm{Y} * \mathrm{H} \tag{3}$$

$$\text{Stiffeners parallel to load dir. } K_s = 0.76 * K + 24.44 * X * H + 8.53 * Y * H - 2.47 * \omega * \beta \tag{4}$$

$$\text{Stiffeners perpendicular to load dir. } K_s = 0.76 * K + 23.41 * X * H + 21.21 * Y * H - 8.35 * \omega * \beta \tag{5}$$

Where
K_o: Elastic buckling coefficient of the plate with circular opening
K : Elastic buckling coefficient of the solid plate $= \left(\frac{\alpha}{\mathrm{m}} + \frac{\mathrm{m}}{\alpha}\right)^2$;
α: W/D; X: OPX/W; Y: OPY/D; H: OPH/D; ω: H_s/W; β: B_s/OPH
m: No. of half waves

Conclusions

The aim of the current study is to investigate the influence of different parameters on the local buckling of steel un-strengthened and strengthened plates with circular openings. These parameters include the plate's aspect ratio, slenderness ratio, circular opening diameter, opening position with respect to the X and Y axes, stiffener dimensions, and configuration. Structural models are established based on the finite element method, where elastic buckling analyses are performed using ANSYS software. Plates with simply supported boundary conditions are subjected to uniaxial compressive uniform pressure. The finite element results are verified against results reported by previous researchers. Comparisons are made considering the different stiffener configurations considered in the current study. The main observations from the study can be summarized as follows:

For unstiffened plates with circular openings:

- The slenderness ratio has no significant effect on the local buckling coefficient.
- For aspect ratios 0.5 and 1.0, the buckling coefficient factor decreases as the opening size increases and the opening location comes near to the center of the plate, and the percentage of decrease reaches 35% of the solid plate strength.
- For aspect ratios 1.5 and 2.0, the buckling coefficient factor decreases when the opening diameter-to-width ratio (OPH/W) is 0.1 for all opening positions. While buckling coefficient ratio starts with values less than one when the opening position with respect to the X-direction to width ratio (OPX/W) is 0.1, then it increases gradually till the opening comes to the center of the plate.

For plates stiffened with stiffeners parallel to load direction:

- For aspect ratios 0.5 and 1.0 and for all opening sizes, the buckling coefficient ratio for stiffened to unstiffened plates with opening (K_s/K_o) increases gradually as the opening moves towards the center of the plate.

- The buckling coefficient ratio for stiffened plates with a circular opening to solid plate (K_s/K_{no}) is greater than 1 for all opening sizes and positions.
- For aspect ratios 1.5 and 2.0, the buckling coefficient ratio for stiffened to unstiffened plates with opening (K_s/K_o) is greater than 1.0 for all opening positions, but it decreases gradually while opening moves towards the center.
- For all aspect ratios except when $\alpha = 1.5$, the change of stiffeners breadth with respect to the opening diameter (B_s/OPH) has a minimum effect on the local coefficient ratio for stiffened to unstiffened plates with opening (K_s/K_o); therefore, it could be neglected.

For plates stiffened with stiffeners perpendicular to load direction:

- For aspect ratios 0.5 and 1.0, the buckling coefficient ratio for stiffened to unstiffened plates with opening (K_s/K_o) is greater than 1.0 for all opening sizes and positions.
- The buckling coefficient ratio of stiffened plates with a circular opening to solid plate ((K_s/K_{no}) becomes less than 1 for all opening sizes and positions, while it becomes greater than 1.0 for aspect ratio equals to 1.0.
- For all aspect ratios, the change of stiffener breadth with respect to the opening diameter (B_s/OPH) has a minimum effect on the local coefficient ratio for stiffened to unstiffened plates with opening (K_s/K_o). Therefore, it could be neglected.

For plates stiffened with box stiffeners:

- For opening moves in X- and Y-directions, the elastic buckling coefficient ratio ($K_{box\ st}/K_o$) increases from 1 to 2.5 as the opening moves towards the center and as the opening size increases.
- When compared to stiffeners parallel or normal to load direction and opening moves in the X-direction, their effect is almost the same with a slight decrease when opening reaches the center of the plate when OPH/W = 0.1–0.3, while it generally increases the strength when OPH/W = 0.4–0.5, but the strength decreases obviously when the opening comes near to center.
- When compared to stiffeners parallel to load direction and opening moves in the Y-direction, stiffeners parallel to the load direction give more strength than box stiffeners for all cases.
- When compared to stiffeners normal to load direction, the elastic buckling coefficient ratio ($K_{box\ st}/K_{vl\ st}$) is less than 1 when OPH/W = 0.1–0.3, while it increases when OPH/W = 0.4–0.5 till it becomes 2.14, but it decreases gradually when the opening comes near to center without being less than 1.

According to the extensive performed parametric study, an empirical formula is derived to estimate the buckling factor coefficient for unstiffened and stiffened steel plates with circular openings.

Abbreviations

B_s	Stiffener breadth
D	Plate height
H_s	Stiffener length

K	Buckling factor
K_{no}	Buckling factor of solid plate
K_o	Buckling factor of an un-strengthened plate with circular opening
K_s	Buckling factor of a strengthened plate with circular opening
LS	Longitudinal stiffener
m	No. of half waves
N_{cr}	Critical buckling load
OPH	Circular opening diameter
OPX	Edge distance from the center of the opening along the Y-axis
OPY	Edge distance from the center of the opening along the X-axis
t_b	Plate thickness
t_s	Stiffener thickness
TS	Transversal stiffener
W	Plate length
α	Plate's aspect ratio
B	Stiffener breadth-to-diameter ratio
λ	Width-to-thickness ratio
σ_{cr}	Critical buckling stress

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Authors' contributions
MM performed the conception, designed the work, aided with the analysis and interpretation of data, and created the first draft. SM performed the literature review, applied analysis, and interpretation of data. MH aided in the conception of work, approved the research work plan, and revised results and writing. AF approved the research approach, design of the work plan, and final version of the thesis and manuscript. The manuscript is a byproduct of a M.SC. supervised and approved by MM, MH, and AF. All authors have read and approved the manuscript.

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